Sample Exam Problems/MA 322, Spring 2012

The first midterm exam takes place February 13, Monday (2:00-3:00pm). It covers Chapter 1.1-1.9, and Chapter 2.1-2.3. There will be five problems. Here are some sample problems that you can practise. The topics that you need to review include:

(1) Elementary row operation and Reduced echlon form of matricx

(2) Solutions of linear system

(3) Linear dependence and linear independence

(4) Linear combination and span of vectors

(5) Linear transformation and its matrix representation

(6) Inverse of square matrix and its application

(7) Applications of linear system.

(1) Use the row operations to determine if the linear system is consistent. If yes, what are the solutions?

$$2x_1 - 4x_4 = -10$$

$$3x_2 + 3x_3 = 0$$

$$x_3 + 4x_4 = -1$$

$$-3x_1 + 2x_2 + 3x_3 + x_4 = 5.$$

(2) Find an equation involve g, h, and k that makes this augmented matrix correspond to a consistent system:

(3) Find the general solutions of the system whose augmented matrix is

$$\begin{bmatrix} 3 & -2 & 4 & 0 \\ 9 & -6 & 12 & 0 \\ 6 & -4 & 8 & 0 \end{bmatrix}$$

(4) For what value of h will y be in the $\text{Span}\{v_1, v_2, v_3\}$ if

$$v_1 = \begin{bmatrix} 1\\ -1\\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 5\\ -4\\ -1 \end{bmatrix}, v_1 = \begin{bmatrix} -3\\ 1\\ 0 \end{bmatrix}, \text{ and } y = \begin{bmatrix} -4\\ 3\\ h \end{bmatrix}$$

(5) Determine if the columns of the matrix A span \mathbf{R}^4 , where A is given by

$$\begin{bmatrix} 4 & -5 & -1 & 8 \\ 3 & -7 & -4 & 2 \\ 5 & -6 & -1 & 4 \\ 9 & 1 & 10 & 7 \end{bmatrix}$$

(6) Find the value(s) of h for which the vectors are linearly dependent. Justify your answer.

1		$\begin{bmatrix} -2 \end{bmatrix}$		3
5	,	-9	,	h
-3		6		-9

(7) Determine if the columns of the matrix form a linearly independent set. Justify your answer.

Γ	-4	-3	0		Γ 1	2	3	2	1
	0	-1	5		1	-2 1	5	2	
	1	1	-5	,	$\begin{vmatrix} -2 \\ 0 \end{vmatrix}$	4	-0 1	2	
	2	1	-10			T	-1	3 -	J

(8) Find the standard matrix associate with the linear map

$$T(x_1, x_2, x_3) = (x_1 - 5x_2 + 4x_3, x_2 - 6x_3).$$

Can a linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^4$ be onto? Can a linear transformation $T : \mathbf{R}^4 \to \mathbf{R}^3$ be one to one?

(9) Let
$$A = \begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix}$$
, $B = \begin{bmatrix} -1 & 1 \\ 3 & 4 \end{bmatrix}$, and $C = \begin{bmatrix} -3 & -5 \\ 2 & 1 \end{bmatrix}$. Verify that $AB = AC$ and yet $B \neq C$.

(10) Determine if $A = \begin{bmatrix} 3 & 6 \\ 4 & 7 \end{bmatrix}$ is invertiable. If yes, find its inverse.

(11) Use the elementary row operation to find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$$