Solutions to Sample Exam Problems

(1)

$$\begin{cases} x_1 = -71/17 \\ x_2 = \frac{45}{17} \\ x_3 = -1-4\cdot\frac{7}{17} = -\frac{45}{17} \\ x_4 = \frac{7}{17} \end{cases}$$

(2)

$$\rightsquigarrow \begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & -3 & 5 & k+2g \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & 0 & 0 & k+2g+h \end{bmatrix}$$

$$\rightsquigarrow \boxed{k+2g+h=0}$$

(3)

$$\rightsquigarrow \begin{bmatrix} 3 & -2 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to 3x_1 - 2x_2 + 4x_3 = 0$$

$$\to \begin{cases} x_1 = \frac{1}{3}(2x_2 - 4x_3) \\ x_2 = x_2 \\ x_3 = x_3 \end{cases}$$

x_2, x_3 free variables

(4)

$$\left[\begin{array}{ccc|c} 1 & 5 & -3 & -4 \\ -1 & -4 & 1 & 3 \\ 2 & -1 & 0 & h \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 5 & -3 & -4 \\ 0 & 1 & -2 & -1 \\ 0 & -11 & 6 & h+8 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 5 & -3 & -4 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & -16 & h-3 \end{array}\right] \rightsquigarrow \{v_1, v_2, v_3\} \text{ is linearly independent}$$

So that it spans all vectors in $\mathbb{R}^3 \longrightarrow h$ is free.

(5) $\sim \begin{bmatrix} -5 & 4 & -1 & 8 \\ -7 & 3 & -4 & 2 \\ -6 & 5 & -1 & 4 \\ 1 & 9 & 10 & 7 \end{bmatrix}$

$$\sim \begin{bmatrix} 0 & 49 & 49 & 43 \\ 0 & 66 & 66 & 44 \\ 0 & 59 & 59 & 46 \\ 1 & 9 & 10 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 9 & 10 & 7 \\ 0 & 49 & 49 & 43 \\ 0 & 66 & 66 & 44 \\ 0 & 59 & 59 & 46 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 9 & 10 & 7 \\ 0 & 1 & 1 & \frac{43}{49} \\ 0 & 1 & 1 & \frac{22}{33} \\ 0 & 1 & 1 & \frac{46}{59} \end{bmatrix} \sim \begin{bmatrix} 1 & 9 & 10 & 7 \\ 0 & 1 & 1 & \frac{43}{49} \\ 0 & 0 & 0 & \frac{22}{33} - \frac{43}{49} \\ 0 & 0 & 0 & \frac{46}{59} - \frac{43}{49} \end{bmatrix}$$

$\rightsquigarrow$ The column are linearly dependent so it can't span $\mathbb{R}^4$.

(P3)

(16) $\begin{bmatrix} 1 & -2 & 3 \\ 5 & -9 & h \\ -3 & 6 & -9 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & h-15 \\ 0 & 0 & 0 \end{bmatrix}$

$\leadsto$ for any ~~vector~~ value of h, the three vectors are L.D.

(17) $\begin{bmatrix} 1 & -2 & 3 & 2 \\ -2 & 4 & -6 & 2 \\ 0 & 1 & -1 & 3 \end{bmatrix}$ the column are L.D. since four vectors in $\mathbb{R}^3$ must be linearly dependent.

$\begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 5 \\ 1 & 1 & -5 \\ 2 & 1 & -10 \end{bmatrix}$ the column are L.I.

(18) $A = \begin{bmatrix} 1 & -5 & 4 \\ 0 & 1 & -6 \end{bmatrix}$

$T: \mathbb{R}^3 \to \mathbb{R}^4$ can't be onto.

$T: \mathbb{R}^4 \to \mathbb{R}^3$ can't be 1-1.

(9). $AB = \begin{bmatrix} -3-18 & 3-24 \\ 1+6 & -1+8 \end{bmatrix} = \begin{bmatrix} -21 & -21 \\ 7 & 7 \end{bmatrix}$

$AC = \begin{bmatrix} -9-12 & -15-6 \\ 3+4 & 5+2 \end{bmatrix} = \begin{bmatrix} -21 & -21 \\ 7 & 7 \end{bmatrix}$.

(10) $\det A = 3\cdot 7 - 4\cdot 6 = -3 \neq 0 \Rightarrow A^{-1}$ exists

$A^{-1} = \frac{1}{-3}\begin{bmatrix} 7 & -6 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} -7/3 & 2 \\ 4/3 & -1 \end{bmatrix}$.

(11) $\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{array}\right]$

$\rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & -2 & 3 & 1 & 0 \\ 0 & -3 & 8 & -2 & 0 & 1 \end{array}\right]$

$\rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & -3 & 8 & -2 & 0 & 1 \\ 0 & 0 & -2 & 3 & 1 & 0 \end{array}\right]$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -8/3 & 2/3 & 0 & -1/3 \\ 0 & 0 & 1 & -3/2 & -1/2 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & -2 & -1 & 0 \\ 0 & 1 & 0 & -10/3 & -4/3 & -1/3 \\ 0 & 0 & 1 & -3/2 & -1/2 & 0 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & -1 & 0 \\ -10/3 & -4/3 & -1/3 \\ -3/2 & -1/2 & 0 \end{bmatrix}$$

$\square$