MA 523/Spring 2017
Homework 1

(Due Thursday, January 26 in class or before 3pm in MATH 714)

1. Let $U \subset \mathbb{R}^n$ be a bounded domain with smooth boundary ∂U. Let ν denote the outward unit normal of ∂U. For two smooth functions f, g on $\overline{U}$. Prove the second Green's identity:
$$\int_U (f\Delta g - g\Delta f) = \int_{\partial U} \big(f\frac{\partial g}{\partial \nu} - g\frac{\partial f}{\partial \nu}\big).$$

2. Write down an explicit formula for a solution u to:
$$\begin{cases} u_t + b \cdot Du + cu = 0 & \text{in } \mathbb{R}^n \times (0, +\infty) \\ u = g & \text{on } \mathbb{R}^n \times \{t = 0\}. \end{cases}$$
Here $c \in \mathbb{R}$ and $b \in \mathbb{R}^n$ are constants.

3. Solve the following equation:
$$au_x + bu_y + cu = 0 \text{ in } \mathbb{R}^2; \; u(x, 0) = f(x) \text{ on } \mathbb{R},$$
where $a, b, c \in \mathbb{R}$ are non-zero constants, and f is a given function.

4. Solve the following equation:
$$u_x - u_y + 5u = e^{3x-4y} \text{ in } \mathbb{R}^2; \; u(x, 0) = 0 \text{ on } \mathbb{R}.$$

5. Prove that Laplace's equation $\Delta u = 0$ is rotation invariant: that is, if O is an orthogonal $n \times n$ matrix and we define
$$v(x) = u(Ox), \; x \in \mathbb{R}^n,$$
then $\Delta v = 0$.