Conference on recent developments in L^∞-variational problems and related areas

University of Kentucky, May 12-14, 2011

TITLE AND ABSTRACT FOR INVITED SPEAKERS

1. Tilak Bhattacharya, Western Ketucky University

TITLE: ON SOLUTIONS TO DIRICHLET PROBLEMS INVOLVING THE INFINITY-LAPLACIAN.
ABSTRACT: Joint work with Ahmed Mohammed. Under appropriate conditions on $f(x; t)$, we prove the existence of viscosity solutions to

$$\Delta_\infty u = f(x; u)$$

that take prescribed continuous data on the boundary of bounded domains. As an application, singular boundary value problems are investigated. These problems are shown to admit viscosity solutions and their asymptotic behavior near the boundary is analyzed. Maximum and comparison principles are used as the main tools in these investigations.

2. Michael Crandall, University of California, Santa Babara

TITLE: Convexity criteria and uniqueness of absolutely minimizing functions.
ABSTRACT: We show that absolutely minimizing functions relative to a convex Hamiltonian $H : \mathbb{R}^n \to \mathbb{R}$ are uniquely determined by their boundary values under minimal assumptions on H. As time permits, we explain the equivalences between comparison with cones, convexity criteria, and absolutely minimizing properties as extended to our generality. This is a joint work with S. N. Armstrong, V.Julin and C. K. Smart.

3. Donatella Danielli, Purdue University

TITLE: Optimal Regularity and the Free Boundary in The Parabolic Signorini Problem.
ABSTRACT: In this talk we present a comprehensive treatment of the parabolic Signorini problem based on a generalization of Almgren's - and Poon's - monotonicity formula for the frequency. Our main result include the proof of the optimal regularity of solutions, the classification of free boundary points, the regularity of the regular set, and the structure of the singular set. This is joint work with N. Garofalo, A. Petrosyan, and T. To.

4. Luigi De Pascale, Universita di Pisa, Italy

TITLE: Existence and duality for L^∞-optimal transport.
ABSTRACT: TBA

5. Peter Juutinen, University of Jyvaskyla, Finland.

TITLE:On the evolution governed by the infinity Laplacian.
ABSTRACT: I will talk about the evolution equation

$$u_t = \Delta_\infty u$$

for the (1-homogeneous version of the) infinity Laplacian

$$\Delta_\infty u = \left(D^2 u \frac{Du}{|Du|}\right)\frac{Du}{|Du|}.$$

I will describe the existence, unqiueness, and regularity for this equation.

6. Robert Jensen, Loyola University of Chicago

TITLE: Δ_∞, Tug of War, and Other Digressions.
ABSTRACT: I intend to re-examine the connection between the Δ_∞ and Game Theory. I will describe the connection between the Armstrong-Smart proof of uniqueness and a viscosity theory proof that the vanishing viscosity solutions of H-J-B equations converge to the Crandall-Lions viscosity solution of the H-J-B equation. I will then consider a variation of the game, introduced and studied by Bob Kohn and Sylvia Serfaty. I will connect this with the recent work of Barron-Goebel-Jensen on quasi-convex (aka level convex) functions.

7. Peter Lindqvist, Norwegian University of Science and Technology, Norway

TITLE: Thoughts about the infinity-eigenvalue problem.
ABSTRACT: The so-called infinity-eigenvalue problem appears as the limit of certain p-eigenvalue problems. A lot is known about this fascinating topic, but a lot remains to be done. The talk is an overview and the emphasis is on open questions.

8. Juan Manfredi, University of Pittsburgh

TITLE: Solutions of Nonlinear PDES in the Sense of Averages.
ABSTRACT: We characterize p-harmonic functions including $p = 1$ and $p = \infty$ by using mean value properties extending a classical result of Privaloff (1925) from the linear case $p = 2$ to all $p's$. We define p-harmonic functions *in the sense of averages* and prove the equivalence of this definition with the viscosity and weak definitions. This is joint work with Mikko Parviainen (Helsinki), and Bernd Kawohl (Cologne).

9. Scott Sheffield, Massachusetts Institute of Technology

TITLE: Optimal Lipschitz extensions of maps to vector spaces and trees: a marriage of conformality and infinite harmonicity, and a game called politics
ABSTRACT: TBA

10. Charles Smart, CIMS, New York University

TITLE: Regularity of infinity harmonic functions
ABSTACT: I will discuss the regularity of infinity harmonic functions, including joint work

with Evans on everywhere differentiability.

11. Peiyong Wang, Wayne State University

TITLE: The Uniqueness/non-uniqueness of the inhomogeneous infinity Laplace equation
ABSTRACT:I will discuss both unqiueness and non-uniqueness of the inhomogeneous infinity Laplace equation

$$\Delta_\infty u := \sum_{i,j} \partial_{x_i} u \partial_{x_j} u \, \partial^2_{x_i x_j} u = f$$

with Dirichlet boundary condition in a bounded domain of $\mathbb{R}^n$.

12. Yifeng Yu, University of California, Irvine

TITLE: Application of L-infinity variational problems in the weak KAM theory.
ABSTRACT: The goal of weak KAM theory is to use PDEs to understand some integrable structures in the Hamiltonian system, e.g, the Aubrey-Mather theory. In this talk, I will give a review of applications of L-infinity variational prolems in the weak KAM theory and present some recent results and open problems.

13.Kaj Nystrom, University of Umea, Sweden

TITLE: Non-Negative p-Harmonic Functions: Regularity and Free Boundary Regularity below the Continuous Threshold.
ABSTRACT: In this talk I will discuss joint work with John Lewis concerning regularity and free boundary regularity below the continous threshold for positive p-harmonic functions, $1 < p < \infty$, vanishing on a portion of a Reifenberg flat or Ahlfors regular NTA-domain. Our results generalize previous works of Kenig and Toro, valid in the case $p = 2$ and for harmonic functions, to the non-linear setting of the p-Laplace operator.

TITLE AND ABSTRACT FOR CONTRIBUTED PRESENTERS

1. David Adams, University of Kentucky

TITLE: The singular set for solutions to systems of strongly coupled quasilinear elliptic PDEs.

ABSTRACT: There are many examples of systems as in the title that can have interior points where the solutions are discontinuous even though the coefficients data are smooth. We give a new way to estimate the size of such sets.

2. Thomas Bieske, University of South Florida

TITILE: The Carnot maximum principle and its application to p-Laplace equations in Carnot groups.

ABSTRACT: In this talk, we present the Carnot maximum principle, which generalizes the Euclidean maximum principle of Crandall-Ishii-Lions. The Carnot maximum principle is then used to show that potential-theoretic weak solutions and viscosity solutions to the p-Laplace equation in Carnot groups coincide.

3. Marian Bocea, North Dakota State University

TITLE: L^∞-Variational Problems and Aronsson Equations from Polycrystal Plasticity.

ABSTRACT: I will indicate some variational principles and the associated Aronsson equations which play a role in the analysis of several models of (first-failure) dielectric breakdown, elec- trical resistivity, and polycrystal plasticity. The corresponding supremal functionals are obtained as Γ-limits of power-law functionals acting on fields subject to constant rank differential constraints.

4. Rafel Goebel, Loyola University of Chicago

TITLE: Viscosity characterizations and convex analysis of quasiconvex and robustly quasiconvex functions.

ABSTRACT: A quasiconvex function is a function whose sublevel sets are convex. A function which is quasiconvex under every linear perturbation is a convex function. A robustly quasiconvex function is a function which is quasiconvex under sufficiently small linear perturbations. The talk will give second-order viscosity characterizations of quasiconvex and robustly quasiconvex functions. Some related uniqueness results for boundary problems will be mentioned. Examples and convex-analytic properties of robustly quasiconvex functions will be presented. This is joint work with E.N. Barron and R.R. Jensen.

5. Jasun Gong, University of Pittsburgh

TITLE: Regularity of Quasi-Minimizers for Non-Homogeneous Energies on Metric Spaces

ABSTRACT: It is well-known that on Euclidean domains, minimizers of the p-Dirichlet energy integral enjoy a rich regularity theory. (For instance, they are Holder continuous and satisfy a Harnack inequality.) Though such functions are solutions to an associated Euler-Lagrange equation (of elliptic type), there exist techniques of proof that rely solely on their energy-minimizing property. In fact, quasi-minimizers – roughly speaking, functions which almost minimize energy – also have similar regularity properties, as shown by Giaquinta

and Giusti in the 1960s. Many notions of analysis, such as Sobolev spaces, extend to the setting of metric spaces equipped with Borel measures. In this setting, we will show that energy quasi-minimizers – both homogeneous and non-homogeneous enjoy similar regularity as their Euclidean counterparts. These results extend the work of J. Kinnunen and N. Shanmugalingam, as well of J. Bjorn, A. Bjorn, and N. Marola. This is based on joint work with J.J. Manfredi and M. Parviainen, and separately with P. Hajlasz.

6. Ben Jaye, University of Missouri, Columbia

TITLE: QUASILINEAR ELLIPTIC EQUATIONS AND SOBOLEV INEQUALITIES.
ABSTRACT: In this talk, we will discuss some joint work with V. G. Maz'ya and I. E. Verbitsky concerning the relationship between certain quasilinear elliptic equations and weighted Sobolev inequalities. For an open set $\Omega \subset \mathbf{R}^n$ and $1 < p < \infty$, we will prove connections between the following weighted L^p-Sobolev inequality, for a distribution $\sigma \in W_{\text{loc}}^{-1, \frac{p}{p-1}}(\Omega)$:

$$|\langle \sigma, |h|^p \rangle| \leq C \int_\Omega |\nabla h|^p dx, \text{ for all } h \in C_0^\infty(\Omega), \tag{0.1}$$

with positive weak solutions to the generalized eigenvalue problem:

$$-\operatorname{div}(|\nabla u|^{p-2}\nabla u) = \sigma |u|^{p-2}u \text{ in } \Omega. \tag{0.2}$$

Furthermore, we will consider weak solutions to the following equation with natural growth in the gradient:

$$-\operatorname{div}(|\nabla v|^{p-2}\nabla v) = (p-1)|\nabla v|^p + \sigma \text{ in } \Omega. \tag{0.3}$$

As a consequence of our results, we characterize the inequality (0.1), thereby obtaining an extension of a result of Maz'ya and Verbitsky in the L^2-case.

7. Teemu Lukkari, University of Jyvaskyla, Finland

TITLE: A minimax problem with a variable exponent.
ABSTRACT: The optimization problem $\min_u \max_x (|\nabla u(x)|^{p(x)}$ has an Euler-Lagrange equation involving the infinity Laplacian. Its solutions are found through a variational procedure, and their unique-ness is derived from the theory of viscosity solutions. This is joint work with P. Lindqvist.

8. Jose Mazon, University of Valencia, Spain

TITLE: On the best Lipschitz extension problem for a discrete distance and the discrete ∞-Laplacian.
ABSTRACT: This lecture is concerned with the best Lipschitz extension problem for a discrete distance that counts the number of steps. We relate this absolutely minimizing Lipschitz extension with a discrete 1-Laplacian problem, which arise as the dynamic programming formula for the value function of some "-tug-of-war games. As in the classical case, we obtain the absolutely minimizing Lipschitz extension of a datum f by taking the limit as p ! 1 in a nonlocal pLaplacian problem.Joint work with J. Rossi and J.J. Toledo.

9. Yelin Ou, Texas A &M University-Commerce

TITLE: ∞-Harmonic maps and morphisms between Riemannian manifolds
ABSTRACT: The talk will begin with how the ∞-harmonic maps between Riemannian manifolds were introduced as a generalization of ∞-harmonic functions and as a limiting case of p-harmonic maps, then go through some examples, properties and constructions of such maps. The second part of the talk will focus on the maps between Riemannian manifolds which preserve the solutions of the ∞-Laplace equations (such a map is called an ∞-harmonic morphism). The characterizations of ∞-harmonic morphisms as horizontally weakly conformal ∞-harmonic maps and as horizontally homothetic maps will be presented. This is a joint work with T. Troutman and F. Wilhelm.

10. Mikko Parviainen, University of Jyvaskyla, Finland

TITLE: Viscosity solutions to $p(x)$-Laplace equation.
ABSTRACT: This talk deals with different notions of solutions to the $p(x)$-Laplace equation

$$- \operatorname{div}(|Du(x)|^{p(x)-2} Du(x)) = 0.$$

It turns out that weak and viscosity solutions are the same class of functions, and that viscosity solutions to Dirichlet problems are unique. As an application, we consider a Radó type removability theorem: if a function $u \in C^1(\Omega)$ is a solution outside its set of zeroes $\{x : u(x) = 0\}$, then it is a solution in the whole domain Ω.

11. Nguyen Phuc, Louisiana State University

TITLE: On Calderón-Zymund theory for p- and $\mathcal{A}$-superharmonic functions.
ABSTRACT: We discuss the boundedness of nonlinear singular operators arising from a class of quasilinear PDEs in divergence form for p- and A-superharmonic functions.

12. Monica Torres, Purdue University

TITLE: ON THE DISTRIBUTIONAL DIVERGENCE OF VECTOR FIELDS VANISHING AT INFINITY.
ABSTRACT: In this talk we present results concerning the solvability of the equation $divv = F$ in various spaces of functions for the vector field v. We find necessary and sufficient conditions on the right hand side F that guarantees the existence of solutions v. We show that the equation $divv = F$ has a solution v in the space of continuous vector fields vanishing at infinity if and only if F belongs to a closed subspace of the dual of $BV_{\frac{m}{m-1}}(\mathbb{R}^m)$ (where the latter is the space of functions in $L^{\frac{m}{m-1}}(\mathbb{R}^m)$ whose distributional gradient is a vector valued measure). In particular we show that, even though $div(\nabla u) = \Delta u = f \in L^m(\mathbb{R}^m)$ need not have a solution $u \in C^1(\mathbb{R}^m)$, to each $f \in L^m(\mathbb{R}^m)$ there corresponds a continuous vector field v vanishing at infinity such that $divv = f$. This is a joint work with Thierry De Pauw.

13. Xiangjun Xu, Binghamton University, SUNY

TITLE: GRADIENT ESTIMATES FOR $u_t = F(u)$ ON MANIFOLDS AND SOME LIOUVILLE-TYPE THEOREMS.
ABSTRACT: In this paper, we first prove a localized Hamilton-type gradient estimate for the positive solutions of Porous Media type equations:

$$u_t = \Delta F(u),$$

with $F(u) > 0$, on a complete Riemannian manifold with Ricci curvature bounded from below. In the second part, we study Fast Diffusion Equation (FDE) and Porous Media Equation (PME):

$$u_t = \Delta(u^p), \, p > 0,$$

and obtain localized Hamilton-type gradient estimates for FDE and PME in a larger range of p than that for Aronson-B enilan estimate, Harnack inequal- ities and Cauchy problems in the literature. Applying the localized gradient estimates for FDE and PME, we prove some Liouville-type theorems for pos- itive global solutions of FDE and PME on noncompact complete manifolds with nonnegative Ricci curvature, which includes Yaus celebrated Liouville Theorem for positive harmonic functions as a special case.

14. Lei Zhang, University of Florida

TITLE: LOCAL GRADIENT ESTIMATE FOR p-HARMONIC FUNCTIONS ON RIEMANNIAN MANIFOLDS.

ABSTRACT: For positive p-harmonic functions on Riemannian manifolds, we derive a gradient estimate and Harnack inequality with constants depending only on the lower bound of the Ricci curvature, the dimension n, p and the radius of the ball on which the function is defined. Our approach is based on a careful application of the Moser iteration technique and is different from Cheng-Yau's method employed by Kostchwar and Ni, in which a gradient estimate for positive p- harmonic functions is derived under the assumption that the sectional curvature is bounded from below. This is a joint work with Xiaodong Wang.

15. JiuyiZhu, Wayne State University

TITLE: An overdetermined problem in Riesz-potential and Bessel-potential.

ABSTRACT: We answer two open questions raised by W. Reichel on characterizations of balls in terms of the Riesz potential and fractional Laplacian to some extent. For a bounded C^1 domain $\Omega \subset \mathbb{R}^N$, we consider the Riesz-potential

$$u(x) = \int_\Omega \frac{1}{|x - y|^{N-\alpha}} \, dy$$

for $2\alpha \neq N$. We show that u =constant on $\partial\Omega$ if and only if Ω is a ball. In the case of $\alpha = N$, the similar characterization is established for the logarithmic potential

$$u(x) = \int_\Omega \log \frac{1}{|x - y|} \, dy.$$

Both conclusions remove W. Reichels question of convexity assumptions Ω. We also prove that such a characterization holds for

$$u(x) = \int |x - y|^{\alpha - N} \log \frac{1}{|x - y|} \, dy$$

when the diameter of the domain Ω is less than $e^{\frac{1}{N-\alpha}}$ in the case when $\alpha - N$ is nonegative even integer, which confirms that such characterization of ball exists for (0.1). This provides a characterization for the overdetermined problem of the fractional Laplacian. We also study such characterizations of ball for Bessel potential integral equation. This is the joint work with Xiaolong Han and Guozhen Lu.