Review Problems for Final Exam: MA 322/Spring 2012

- The final exam will take place 8:00-10:00 am, Friday May 4, CB 341.
- The content includes Chapter 1– Chapter 5 (all those sections covered in classes).
- The below is a list of review problems.

The topics you need to review include:

(1) Row operations to transform matrices into reduced echelon forms.
(2) Solutions to $AX = b$.
(3) Col(A) and Nul(A).
(4) Relationship between A and linear transformation.
(5) Inverse of matrix and its characterization and row operation to find A^{-1}.
(6) Partitioned matrix
(7) LU factorization
(8) Leontiff Input-Output model
(9) Subspaces of $\mathbb{R}^n$: basis, dimension
(10) Rank of matrix
(11) Determinants and their properties
(12) Cramer's rule
(13) Area and volume and their relations with determinant.
(14) Eigenvalues and eigenvectors of square matrices and linear transformations.
(15) Diagonalization of matrices.
(16) Complex eigenpairs.
(17) Application to system of first linear ODE.

1. Use the row operation to solve the system

$$\begin{aligned} x_1 + 4x_2 - 2x_3 + 8x_4 &= 12 \\ x_2 - 7x_3 + 2x_4 &= -4 \\ 5x_3 - x_4 &= 7 \\ x_3 + 3x_4 &= -5. \end{aligned}$$

2. Is b is in the span of a_1, a_2, a_3, where

$$a_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, a_2 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}, a_3 = \begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix}, b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$$

3. Determine if the column of the matrix

$$A = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 5 & 8 & 0 \end{bmatrix}$$

are linearly independent.

4. Without using row reduction, find the inverse of the partitioned matrix

$$A = \begin{bmatrix} 3 & 2 & 0 & 0 & 0 \\ 5 & 3 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 10 & 7 \\ 0 & 0 & 0 & 7 & 5 \end{bmatrix}$$

5. Find the inverse of

$$A = \begin{bmatrix} 1 & -2 & -1 \\ -1 & 5 & 6 \\ 5 & -4 & 5 \end{bmatrix}$$

by the row operation, if it exists.

6. Find the LU factorization of the matrix

$$A = \begin{bmatrix} 1 & -2 & -2 & -3 \\ 3 & -9 & 0 & -9 \\ -1 & 2 & 4 & 7 \\ -3 & -6 & 26 & 2 \end{bmatrix}$$

Then use the LU factorization to find the solution of

$$Ax = \begin{bmatrix} 1 \\ 6 \\ 0 \\ 3 \end{bmatrix}.$$

7. Solve the Leontiff production equation for an economy with three sectors, given that

$$C = \begin{bmatrix} 0.1 & 0.2 & 0 \\ 0.3 & 0.4 & 0.2 \\ 0.1 & 0 & 0.2 \end{bmatrix} \quad \text{and} \quad \mathbf{d} = \begin{bmatrix} 40 \\ 60 \\ 80 \end{bmatrix}$$

8. Find the rank of A and the dimension of Nul (A), where

$$A = \begin{bmatrix} 1 & 3 & 2 & -6 \\ 3 & 9 & 1 & 5 \\ 2 & 6 & -1 & 9 \\ 5 & 15 & 0 & 14 \end{bmatrix}$$

9. Determine whether the following statement is true or false. If it is true, then give a reason. If it is false, give an example.

a) Can it be true that there exist five vectors in R^4 that are linearly independent?

b) For three 2×2 matrices A and B and C, $AB = AC$ implies $B = C$.
c) For a 6×9 matrix A, is it possible that rank of A equals 5, and the Nul(A) has dimesion 3?
d) the columns of A are linearly independent but $\det(A) = 0$.
e) Is it possible that 2 is an eigenvalue of A with multiplicity 3, while its eignspace has dimension 2?

10. Calculate the determinant of the matrix

$$A = \begin{bmatrix} 6 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 2 & -6 & -7 & 5 \\ 5 & 1 & 4 & 3 \end{bmatrix}$$

11. Combine the methods of row reduction and cofactor expansion to compute the determinat of

$$A = \begin{bmatrix} 1 & a & a^2 & a^3 \\ 1 & b & b^2 & b^3 \\ 1 & c & c^2 & c^3 \\ 1 & d & d^2 & d^3 \end{bmatrix}$$

where a, b, c, d are four real numbers.
12. Use the Cramer's rule to solve the linear system

$$\begin{aligned} 2x_1 + x_2 + x_3 &= 1 \\ -x_1 + \quad 2x_3 &= 3 \\ 3x_1 + x_2 + 3x_3 &= 5 \end{aligned}$$

13. Find all eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 6 & 2 & 0 & 0 \\ 0 & 3 & 6 & 0 \\ 2 & 3 & 3 & -5 \end{bmatrix}$$

Then find P and D such that $A = PDP^{-1}$.
14. Diagonlize the matrix, if possible.

$$A = \begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$$

15. Find the eigenvalue and eigenvector of

$$A = \begin{bmatrix} -11 & -4 \\ 20 & 5 \end{bmatrix}$$