Review Problems for Midterm I, MA213, Fall 2013

Exam Date: Wednesday,October 9 2013

- The first midterm includes chapter 12.1-12.7, chapter 13.1-13.5, and chapter 14.1-14.4.
- The time of exam is: 2:00-2:50 pm.
- The place of exam is: CB 106

Here is a set of review problems.

1. Compute the distance between $\mathbf{A} = (2, -1, 7)$ and $\mathbf{B} = (1, -3, 5)$.

2. Calculate the angle between the two vectors

$$\mathbf{a} = \langle 2, -3, 1\rangle,\ \mathbf{b} = \langle 1, 6 - 2\rangle.$$

3. Calculate the area of the parallelogram formed by the two edges **AB** and **AC**, where

$$\mathbf{A} = (1, 3, 4),\ \mathbf{B} = (2, 7, -5),\ \mathbf{C} = (-1, 0, 3).$$

4. Calculate the volume of the parallelopepid formed by the adjacent edges **PR, PQ, PS**, where

$$\mathbf{P} = (2, 0, -1),\ \mathbf{Q} = (4, 1, 0),\ \mathbf{R} = (3, -1, 1),\ \mathbf{S} = (2, -2, 2).$$

5. Calculate the distance from the point $\mathbf{P} = (1, 3, 2)$ to the plane

$$4x + 5y + 6z + 7 = 0.$$

6. Find the equation of the line passing through two points

$$\mathbf{P} = (1, 2, 3),\ \mathbf{Q} = (-2, -1, -3).$$

7. Find the equation of the plane passing through two lines:
L1: $x = t + 1,\ y = 2t + 2,\ z = 3t + 3,$
L2: $x = -2t + 4,\ y = 3t - 3,\ z = -t.$

8.
a) Suppose the rectangle coordinate of a point **P** is $(1, \sqrt{3}, 2\sqrt{3})$. Calculate

both its cylindrical coordinate and spherical coordinate.
b) Suppose the spherical ordinate of **Q** is $(2\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{3})$. Find its rectangle coordinate and cylindrical coordinate.

9.
a) Calculate the velocity and acceleration of a particle whose position function is

$$\mathbf{r}(t) = (t,\ t^2,\ t^3).$$

b) Find the position function of a particle if its initial position is $\mathbf{r}(0) = \langle 1, -1, 2\rangle$, initial velocity is $\mathbf{v}(0) = \langle -2, 0, 3\rangle$, and the acceleration is

$$\mathbf{a}(t) = \langle -2t,\ 3t^3,\ e^t\rangle.$$

10. Calculate the arc length of the curve

$$\mathbf{r}(t) = (e^t,\ e^t \sin t,\ e^t \cos t), \quad 0 \le t \le 2\pi.$$

11. Calculate the curvature of the curve

$$\mathbf{r}(t) = (e^{-t},\ 2t,\ \ln(1+t^2)).$$

12. Calculate the unit tangent vector **T** and the normal vector **N** of the curve

$$\mathbf{r}(t) = (\sqrt{2}\cos t,\ \sin t,\ \sin t).$$

13. Let $f(x,y) = \dfrac{x^3+y^3}{x^2+y^2}$ for $(x,y) \neq (0,0)$. Find $\lim\limits_{(x,y)\to(0,0)} f(x,y)$.

14. Find the first order partial derivatives of $f(x,y) = \dfrac{x}{\sqrt{x^2+y^2}}$ for $(x,y) \neq (0,0)$.

15. Find the linear approximation of $f(x,y) = \sqrt{9-4(x^2+y^2)}$ at $(x,y) = (1,1)$ and use it to approximate $f(1.01, 0.99)$.

16. Estimate $f(1.02.0.01, -0.03)$ assuming that

$$f(1,0,0) = -3, \quad f_x(1,0,0) = -2, \quad f_y(1,0,0) = 4, \quad f_z(1,0,0) = 2.$$