Review Problems for Midterm II, MA 213, Fall 2013

Exame Date: Wednesday November 13 2013

- The second midterm exam coveres Chapter 14.5-14.7, and Chapater 15.1-15.5.
- Time and Place: 2:00-2:50 pm, CB 106.

Here is a set of review problems.

1. Find an equation of the tangent plane to the surface $x^2 + z^2 e^{y-x} = 13$ at the point $P = \left(2, 3, \frac{3}{\sqrt{e}}\right)$.

2. Calculate the directional derivative in the direction $\mathbf{v}$ at the given point P for $f(x, y, z) = x\ln(y+z)$, $\mathbf{v} = (2, -1, 1)$, and $P = (2, e, e)$.

3. Use the chain rule to calculate the partial derivatives: $\frac{\partial h}{\partial q}$ at $(q, r) = (3, 2)$, where $h(u, v) = ue^v$, $u = q^3$, $v = qr^2$.

4. Use implicit differentiation to calculate the partial derivative: $\frac{\partial w}{\partial z}$, where $x^2 w + w^3 + wz^2 + 3yz = 0$.

5. Find the critical points of the function, Then use the Second Derivative Test to determine whether they are local minima, local maxima, or saddle points: $f(x, y) = x^3 + y^4 - 6x - 2y^2$; $g(x, y) = \ln x + 2\ln y - x - 4y$.

6. Determine the global extreme values of the function on the given domain: $f(x, y) = (4y^2 - x^2)e^{-x^2-y^2}$, $x^2 + y^2 \leq 2$.

7. Calculate the double integral

$$\int\int_R (xy^2 + \frac{y}{x})\, dA$$

where

$$R = \{(x, y) | 2 \leq x \leq 3, -1 \leq y \leq 0\}.$$

8. Use the polar coordinate to calculate the double integral

$$\int_0^1 \int_y^{\sqrt{1-y^2}} \frac{1}{3 + x^2 + y^2}\, dx\, dy.$$

9. Evaluate the double integral

$$\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} x^2 y^2\, dx dy.$$

10. Calculate the volume of the region above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.

11. Find the mass of the region D that is enclosed by the cardioid $r = 1 + \cos\theta$ with density $\rho(x, y) = \sqrt{x^2 + y^2}$.

12. Use the Fubini's theorem (or equivalently, the iterated integration) to evulate the triple integral

$$\int\int\int_E yz\cos(x^5)\,dV,$$

where

$$E = \Big\{(x,y,z) \mid 0 \le 1 \le 1,\ 0 \le y \le x,\ 0 \le z \le 2x\Big\}.$$

13. Use the spherical coordinates to calculate

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-y^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2\sqrt{x^2+y^2+z^2}\,dzdxdy.$$

14. Find the center of mass for the lamina that occupies the region D and has the given density function ρ: D is the triangular region with vertices $(0,0)$, $(2,1)$, $(0,3)$; $\rho(x,y) = x + y$.

15. Evaluate the double integral by making an appropriate change of variables

$$\int\int_{\mathbf{R}} \frac{x+2y}{\cos(x-y)}\,dxdy,$$

where $\mathbf{R}$ is the parallelogram bounded by the lines $y = x,\ y = x-14,\ x+2y = 0,\ x+2y = 2$.

16. Use the map

$$G(u,v) = \big(\frac{u+v}{2},\ \frac{u-v}{2}\big)$$

to compute

$$\int\int_{\mathcal{R}} ((x-y)\sin(x+y))^2\,dxdy,$$

where $\mathcal{R}$ is the square with vertices $(\pi,0)$, $(2\pi,\pi)$, $(\pi,2\pi)$, and $(0,\pi)$.