Sample Exam Problems/MA 214, Spring 2012

The first midterm exam takes place February 10, Friday (1:00-2:00pm). It covers Chapter 1, 2, and Chapter 3. There will be five problems. Here are some sample problmes that you can practise. The topics that you need to review include:
(1) How to solve first order linear ODE?
(2) How to solve first order separable ODE?
(3) How to check a first order ODE is exact? If yes, how to solve it? How to find the integrating factors in the two simple case if the equation is not exact, then solve it?
(4) How to solve second order constant coefficient linear ODEs when the characteristic equation has two distinct real roots?
(5) Applications to autonomous equation and population models?

(1) Find the solution to the initial value problem of the differential equation:

$$\begin{aligned} y' + \frac{2}{t}y &= \frac{1+\cos t}{t^2} \\ y(\frac{\pi}{2}) &= 1. \end{aligned}$$

(2) First find the solution to the initial value problem of the differential equation:

$$\frac{dy}{dx} = \frac{1+3x^2}{3y^2-6y}, \ y(0) = 1.$$

(3) Consider the differential equation:

$$\frac{dy}{dt} = y(4-y).$$

First determine the stable and unstable equilibrium solutions. Then find general solutions of the equation. (Hint: use the partial fraction method $\frac{1}{y(4-y)} = \frac{A}{y} + \frac{B}{4-y}$)).

(4) First verify that the differential equation

$$(e^x \sin y - 2y \sin x + x^2)\, dx + (e^x \cos y + 2 \cos x + y^2)\, dy = 0$$

is exact. Then find a general solution of the differential equation.

(5) A $200m^3$ tank is full of water that contains a pollutant Z at a concentration of $0.5g/m^3$. Cleaner water, with a pollutant concentration of $0.2g/m^3$ is pumped into the well-mixed tank at a rate of $5m^3/sec$. Water flows out of the tank through an overflow valve at the same rate as it is pumped in. Determine the amount and concentration of pollutant in the tank as a function of the time elapsed since cleaned water is pumped.

(6) Study the example in Chapter 2.5, page 83 of the book.

(7) Find a general solution to the differential equation:

$$y'' - 3y' + 2y = 0.$$