Some Review Problems for Midterm III, MA 214, Spring 2012

This exam covers Chapter 4.1, 4.2, 4.3, 4.4. Chapter 5.1, 5.2. Chapter 6.1, 6.2. It will take place Friday, March 13, and it will consist of six problems. Below is a set of review problems.

(1). State the definition of linear dependence(or independence) of n-functions: $f_1(t), \cdots, f_n(t)$. Use the Wroskian method to check that $y_1(t) = e^t$, $y_2(t) = e^{2t}$, and $y_3(t) = e^{3t}$ are linearly independent.

(2). Find a general solution to the following equation:

$$y^{(4)} + y = 0.$$

(3). Solve the initial value problem for

$$\begin{aligned} 5y''' + 6y'' + y' &= 0 \\ y(0) &= -1 \\ y'(0) &= 1 \\ y''(0) &= 2. \end{aligned}$$

(4). Use the undertermined method to find the right form of a particular solution to

$$y^{(4)} - 8y^{(2)} + 16y = e^{2t}.$$

(5). Determine a suitable form for $Y(t)$ by the method of undetermined coefficients for the following equation:

$$y^{(4)} + 2y''' + 2y'' = 3e^t + (2t^2)e^{-t} + te^{-t}\sin t.$$

(6). Verify that $y_1(x) = x$, $y_2(x) = x^2$, and $y_3(x) = x^3$ are three linearly independent solutions to the homogeneous equations of

$$x^3y''' - 3x^2y'' + 6xy' - 6y = x^3g(x)$$

Then use the variation of parameter method to find the formular of a particular solution to the nonhomogeneous equation (Hint: don't forget to change the equation into the standard form first).

(7). Find a series solution of the equation

$$xy'' + y' + xy = 0, \ x_0 = 1.$$

(8). Find the first five terms in the series solution of the following initial value problem

$$(2 + x^2)y'' - xy' + 4y = 0, \ y(0) = -1, \ y'(0) = -3.$$

(9). Find the Laplace transform of the following functions

$$t \sin t, \ te^{at}, \ t^n e^{at}, t^{\frac{1}{2}}.$$

(10). Find the inverse Laplace transform of the following functions by the paritial fraction method

$$\frac{8s^2 - 4s + 12}{s(s^2 + 4)}, \ \frac{2s - 3}{s^2 + 2s + 10}.$$

(11). Find the solution of the initial value problem by the Laplace transform

$$y'' + \omega^2 y = \cos 2t, \ \omega^2 \neq 4, \ y(0) = 1, \ y'(0) = -1.$$

(12). Find the solution of the initial value problem by the Laplace transform

$$y^{(4)} - 4y''' + 6y'' - 4y' + y = 0, \ y(0) = 0, \ y'(0) = 1, \ y''(0) = 0, \ y'''(0) = 1.$$