Review Problems for Final Exam, MA 214, Spring 2012

- The final exam covers Chapter 2, Chapter 3, Chapter 4, Chapter 5.1, 5.2, and Chapter 6.
- It will take place Monday April 30, 8-10 am at CB 339.
- It will consist of 10 problems.

The following is a list of review problems that may help you to prepare for the final exam.

(1). Use the integral factor method to solve the initial value problem

$$\begin{aligned} (1+t^2)y' + 4ty &= (1+t^2)^{-2} \\ y(0) &= 1 \end{aligned}$$

(2). Solve the separable equation

$$\begin{aligned} y^2(1-x^2)^{\frac{1}{2}}dy &= \arcsin x dx \\ y(0) &= 1 \end{aligned}$$

(3). Verify that the following equation is exact. Then solve it.

$$(ye^{xy}\cos 2x - 2e^{xy}\sin 2x + 2x)\,dx + (xe^{xy}\cos 2x - 3)\,dy = 0.$$

(4). Suppose that a certain population obeys the logistic equation

$$\frac{dy}{dt} = ry[1-(y/K)]$$

(a) If $y_0 = \frac{K}{3}$, find the time τ at which the initial population has doubled. Find the value of τ corresponding to $r = 0.025$ per year.
(b) If $y_0/K = \alpha$, find the time T at which $y(T)/K = \beta$, where $0 < \alpha, \beta < 1$. Observe that $T \to \infty$ as $\alpha \to 0$ or as $\beta \to 1$. Find the value of T for $r = 0.025$ per year, $\alpha = 0.1$, and $\beta = 0.9$.

(5). State the definition of linear dependence(or independence) of n-functions: $f_1(t), \cdots, f_n(t)$. Use the Wronskian method to check that $y_1(t) = e^{\lambda_1 t}$,

$y_2(t) = e^{\lambda_2 t}$, and $y_3(t) = e^{\lambda_3 t}$ are linearly independent, provided that $\lambda_1, \lambda_2, \lambda_3$ are distinct real values.

(6). Use the characteristic method to find a general solution to the following equation:

$$y^{(8)} - y = 0.$$

(7). Use the Laplace transformation method to solve the initial value problem for

$$\begin{aligned} y''' - 3y'' + 2y' &= t + e^t \\ y(0) &= 1 \\ y'(0) &= -1 \\ y''(0) &= -\frac{3}{2}. \end{aligned}$$

(8). Use the undertermined coefficient method to find the right form of a particular solution to

$$y^{(4)} + 2y^{(3)} + 2y'' = 3e^t + te^{-t} + e^{-t}\sin t.$$

(9). Use the variation of parameter method to find a formula involving integrals for a particular solution of the differential equation

$$y''' - y'' + y' - y = g(t)$$

(10). Find a series solution of the equation

$$y'' + xy' + 5y = 0, \ y(0) = 1, y'(0) = -1.$$

(11). Find the first five terms in the series solution of the following initial value problem

$$(1 + x)y'' - xy' + 4x^2 y = 0, \ y(0) = 1, \ y'(0) = 3.$$

(12). Find the Laplace transform of the following functions

$$e^{2t}\sinh(3t), \ u_1(t)t^n, \ t^2 e^t \cos t.$$

(13). Find the inverse Laplace transform of the following functions by the paritial fraction method

$$\frac{8s^2 - 4s + 12}{(s-1)(s+2)(s^2+1)}, \quad \frac{2s+5}{s^2+2s+17}.$$

(14). Find the solution of the initial value problem by the Laplace transform

$$y'' + y' + \frac{5}{4}y = 1 - u_\pi(t)\sin t, \;\; y(0) = 1, \;\; y'(0) = -1.$$

(15). Find the solution of the initial value problem by the Laplace transform

$$y'' + 4y = g, \;\; y(0) = 0, \;\; y'(0) = 0,$$

where

$$g(t) = 0, 0 < t < 5; \;\; = \frac{t-5}{5}, 5 \le t < 10; \;\; = 1, t \ge 10.$$