Review Problems for the First Midterm of MA 213, Calculus III

Exam Date: Wednesday, 10/12/2011

The first midterm includes Chapters 12, 13, 14.
The time of exam is: 12:55 pm-1:50pm.
The place of exam is: CB 349.

Here is a set of some review problems.

1. Compute the distance between $A = (2, -1, 7)$ and $B = (1, -3, 5)$.

2. Calculate the angle between the two vectors

$$\mathbf{a} = \langle 2, -3, 1 \rangle, \ \mathbf{b} = \langle 1, 6 - 2 \rangle.$$

3. Calculate the area of the parallelogram formed by the two edges **AB** and **AC**, where

$$\mathbf{A} = (1, 3, 4), \ \mathbf{B} = (2, 7, -5), \ \mathbf{C} = (-1, 0, 3).$$

4. Calculate the volume of the parallelopepid formed by the adjacent edges PR, PQ, PS, where

$$P(2, 0, -1), \ Q(4, 1, 0), \ R(3, -1, 1), \ S(2, -2, 2).$$

5. Calculate the distance from the point $P = (1, 3, 2)$ to the plane

$$4x + 5y + 6z + 7 = 0.$$

6. Find the equation of the line passing through two points

$$P = (1, 2, 3), \ Q = (-2, -1, -3).$$

7. Find the equation of the plane passing through two lines:
L1:

$$x = t + 1, \ y = 2t + 2, \ z = 3t + 3,$$

L2:

$$x = -2t + 4, \ y = 3t - 3, \ z = -t.$$

8. Calculate the arc length of the curve

$$\mathbf{r}(t) = (e^t, \ e^t \sin t, \ e^t \cos t), \quad 0 \le t \le 2\pi.$$

9. Calculate the curvature of the curve

$$\mathbf{r}(t) = (e^{-t},\ 2t,\ \ln(1+t^2)).$$

10. Calculate the unit tangent vector T and the normal vector N of the curve

$$\mathbf{r}(t) = (\sqrt{2}\cos t,\ \sin t,\ \sin t).$$

11. Does the limit

$$\lim_{(x,y)\to(0,0)} \frac{6x^3y}{2x^4+y^4}$$

exist? If yes, then prove your answer by the definition of limit. If not, then indicate your reason.

12. Use the implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ of $yz^4 + x^2z^3 = e^{xyz}$.

13. Find the equation of tangent plane and normal line to the surface

$$\sin(xyz) = x + 2y + 3z, \text{ at the point } (2,-1,0).$$

14. Calculate the first partial derivatives of the function

$$f(x,y,z) = \sqrt{x^2+y^2}z^2.$$

Then use it to approximate the number $\sqrt{(2.99)^2+(3.99)^2}\,(1.01)^2$.

15. Find the maximum rate of change of f at the given point:

$$f(x,y,z) = \tan(x+2y+3z),\ (-5,1,1).$$

15. Find all critical points of

$$f(x,y) = x^2 - xy + y^2 + 9x - 6y + 10,$$

and then determine their types (i.e.,local maximum and minimal points or saddle points).

16. Find the absolute maximum and minimum values of the function

$$f(x,y) = 4x + 6y - x^2 - y^2, \text{ on } D = \{(x,y) \mid 0 \le x \le 4, 0 \le y \le 5\}.$$

17. Use the Lagrange multiplier method to find the maximum and minimum of

$$f(x,y) = xy$$

subject to the given constraints

$$x^2 + y^2 = 1.$$