Review Problems for Second Midterm Exam: MA 322/Spring 2012

The second midterm exam will take place 1:55pm–2:55pm, Friday, March 23.

The sections to be covered include: Chapter 2.4, 2.5, 2.6, 2.8, 2.9, and Chapter 3.1, 3.2, 3.3.

The below is a list of review problems. The topics you need to review include:

(1) Partitioned matrix

(2) LU factorization

(3) Leontiff Input-Output model

(4) Subspaces of $\mathbb{R}^n$: basis, dimension

(5) Rank of matrix

(6) Determinants and their properties

(7) Cramer's rule

(8) Area and volume and their relations with determinant.

1. Without using row reduction, find the inverse of the partitioned matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 3 & 5 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 7 & 8 \\ 0 & 0 & 0 & 5 & 6 \end{bmatrix}$$

2. Find the LU factorization of the matrix

$$A = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 4 & 4 \\ 0 & 2 & 1 \end{bmatrix}$$

Then use the LU factorization to find the solution of

$$Ax = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}.$$

3. Solve the Leontiff production equation for an economy with three sectors, given that

$$C = \begin{bmatrix} 0.2 & 0.2 & 0 \\ 0.3 & 0.1 & 0.3 \\ 0.1 & 0 & 0.2 \end{bmatrix} \quad \text{and} \quad \mathbf{d} = \begin{bmatrix} 40 \\ 60 \\ 80 \end{bmatrix}$$

4. Let

$$A = \begin{bmatrix} 1 & -1 & 5 \\ 2 & 0 & 7 \\ -3 & -5 & -3 \end{bmatrix}$$

and

$$\mathbf{u} = \begin{bmatrix} -7 \\ 3 \\ 2 \end{bmatrix}$$

Is $\mathbf{u}$ in Nul A? Is $\mathbf{u}$ in Col A?

5. Find the matrix C whose inverse $C^{-1} = \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix}$.

6. a) State the definition of the basis and the dimension of a subspace H of the vector space $\mathbb{R}^n$.

b) State the definition of the rank of $m \times n$ matrix A.

7. Determine whether the following statement is true or false. If it is true, then give a reason. If it is false, give an example.

a) An 4×4 matrix can be invertible while its column vectors do not span $\mathbb{R}^4$.

b) For two 2×2 matrices A and B, $AB = 0$ implies either $A = 0$ or $B = 0$.

c) For two 3×3 matrices A and B, if AB is invertiable, then so does A.

d) For a square matrix A, $\det(A^3) = (\det A)^3$.

e) If B is a square matrix and $B^T B = I$, then $\det(B) = \pm 1$.

f) If B is a square matrix of order $n \geq 2$, then $\det(5B) = 5\det(B)$.

g) If the columns of A are linearly dependent, then $\det(A) = 0$.

8. Calculate the determinant of the matrix

$$A = \begin{bmatrix} 1 & -2 & 5 & 2 \\ 0 & 3 & 0 & 0 \\ 2 & -6 & -7 & 5 \\ 5 & 1 & 4 & 3 \end{bmatrix}$$

9. Combine the methods of row reduction and cofactor expansion to compute the determinat of

$$A = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$$

where a, b, c are three real numbers.

10. Use the Cramer's rule to solve the linear system

$$\begin{aligned} 2x_1 + x_2 + x_3 &= 4 \\ -x_1 + \quad 2x_3 &= 2 \\ 3x_1 + x_2 + 3x_3 &= -2 \end{aligned}$$

11. Find the inverse of the matrix by using the Cramer's rule

$$A = \begin{bmatrix} 3 & 6 & 7 \\ 0 & 2 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$