(P1)

Solution to 1st Midterm Review Problem

(1) $\mu(t) = e^{\int \frac{2}{t} dt} = t^2$

$(t^2 y)' = 1 + \cos t$

$\int_{\pi/2}^{t} (t^2 y)' dt = \int_{\pi/2}^{t} (1 + \cos t) dt = t - \frac{\pi}{2} + \sin t - \sin\left(\frac{\pi}{2}\right)$

$\qquad \parallel$

$t^2 y(t) - \left(\frac{\pi}{2}\right)^2 \cdot 1 = t - \frac{\pi}{2} + \sin t - 1$

$\Rightarrow \quad y(t) = \frac{1}{t^2}\left[\frac{\pi^2}{4} + (t - 1) + \sin t - \frac{\pi}{2}\right]$ □

(2) $\int_0^{x} (3y^2 - 6y)\, dy = \int_0^{x} (1 + 3x^2)\, dx = x + x^3$

$\Downarrow$

$y^3 - 3y^2 \Big|_{x=0}^{x=x} = x + x^3$

$\Downarrow$

$y^3 - 3y^2 - (1^3 - 3 \cdot 1^2) = x + x^3$

$\Downarrow$

$\boxed{y^3 - 3y^2 = x + x^3 - 2}$

P2

(3) $\int \frac{dy}{y(4-y)} = \int dt = t + c$

$$\int \frac{1}{y(4-y)} = \frac{1}{4}\int \left(\frac{1}{y} + \frac{1}{4-y}\right) = \frac{1}{4}\left[\ln(y) - \ln(4-y)\right]$$

$$= \frac{1}{4}\ln\left|\frac{y}{4-y}\right| \Rightarrow \boxed{\frac{y}{4-y} = ce^{4t}}$$

as $t \to \infty$ $\quad y \to 4$ $\quad \begin{cases} y \equiv 0 & \text{ansstable ;} \\ y \equiv 4 & \text{asymptotically stable.} \end{cases}$

(4) $M_y = e^x \cos y - 2\sin x$

$N_x = e^x \cos y - 2\sin x$

$M_y = N_x \Rightarrow$ It is exact

$$F = \int (e^x \sin y - 2y\sin x + x^2)\,dx + G(y)$$

$$= e^x \sin y + 2y\cos x + \frac{x^3}{3} + G(y)$$

$$F_y = e^x \cos y + 2\cos x + G'(y) = e^x \cos y + 2\cos x + y^2 \Rightarrow$$

P3

$G'(y) = y^2$ or $G(y) = \frac{y^3}{3}$.

Hence the solution is:

$$\boxed{e^x \cos y + 2\cos x + \frac{x^3}{3} + \frac{y^3}{3} = \text{const}}$$

(5) and (6): Skipped

(7). Char. eq'n. is: $\lambda^3 - 3\lambda + 2 = 0$

has two roots $\lambda_1 = 1$, $\lambda_2 = 2$

$$\Rightarrow \boxed{y_c = c_1 e^t + c_2 e^{2t}}$$