AVE 85.2 A 88-100 B 65-87 C 50-64

MA 527 **Exam 2** **Fall 2009** **Name** ______________________

1. (15) Find the Laplace transform $F(s)$ of $f(t) = tu(t-1)$.

$$e^{-s}\mathcal{L}\{t+1\} = e^{-s}\left(\frac{1}{s^2}+\frac{1}{s}\right)$$

F(s)= $e^{-s}\left(\frac{1}{s^2}+\frac{1}{s}\right)$

2. (15) Find the inverse Laplace transform $f(t)$ of $F(s) = \dfrac{se^{-s}}{(s+3)^2+4}$.

$$\frac{(s+3)e^{-s}}{(s+3)^2+4} - \frac{3e^{-s}}{(s+3)^2+4}$$

$$u(t-1)e^{-3(t-1)}\sin 2(t-1) - u(t-1)e^{-3(t-1)}\frac{3}{2}\cos 2(t-1)$$

$f(t) = u(t-1)e^{-3(t-1)}\left(\cos 2(t-1) - \frac{3}{2}\sin 2(t-1)\right)$

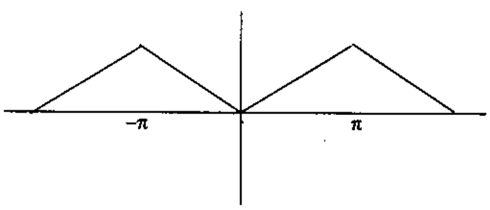
3.(15) Match the following Fourier series with appropriate pictures without computing Fourier series. Give reasons for your choices.

i) $2\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin nx$ D odd function

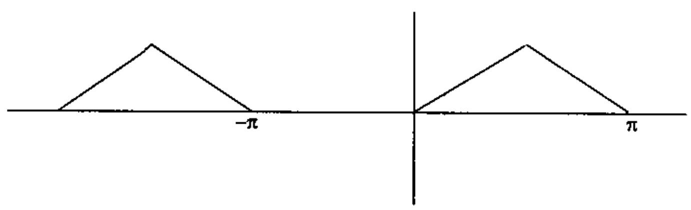
ii) $\frac{\pi}{2}-\sum_{n=1}^{\infty}\frac{1}{n}\sin nx$ $\pi/2$ at $x=0$

iii) $\frac{\pi}{2}-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n+1)^2}\cos(2n+1)x.$ A even function

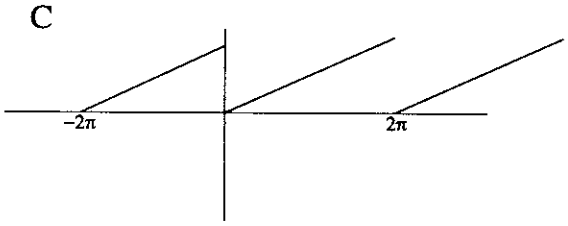
A



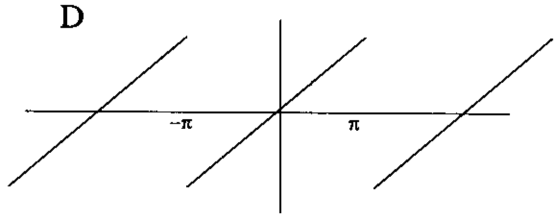
B



C



D



4. (15) Given that the Fourier sine series for $f(x) = x(\pi - x) \quad 0 \le x \le \pi$ is

$$f(x) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1-(-1)^n}{n^3} \sin nx,$$

use Parseval's identity to compute the sum $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(2n+1)^6}$.

$$f(x) = \frac{8}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^3} \sin(2n+1)x$$

$$\frac{2}{\pi}\int_0^{\pi} x^2(\pi-x)^2\,dx = \frac{64}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^6}$$

$$\frac{2}{\pi}\int_0^{\pi} \pi^2 x^2 - 2\pi x^3 + x^4\,dx = \frac{2}{\pi}\left(\frac{\pi^5}{3} - \frac{\pi^5}{2} + \frac{\pi^5}{5}\right)$$

$$= 2\pi^4\left(\frac{10-15+6}{30}\right) = \frac{\pi^4}{15}$$

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^6} = \frac{\pi^4}{15} \cdot \frac{\pi^2}{64}$$

64
15
320
64

$$\boxed{\pi^6/960}$$

5a. (15) Let

$$f(x) = \begin{cases} 1 & 0 < x < 2\pi \\ 0 & \text{otherwise.} \end{cases}$$

Compute the Fourier cosine transform of f.

$$\frac{\sqrt{2}}{\sqrt{\pi}} \int_0^{2\pi} \cos wx \, dx = \sqrt{\frac{2}{\pi}} \frac{1}{w} \sin 2\pi w$$

Fourier cosine transform

$$\boxed{\sqrt{\frac{2}{\pi}} \frac{1}{w} \sin 2\pi w}$$

5b. (10) Using Problem 5a above, evaluate

$$f(\pi) = \frac{2}{\pi} \int_0^{\infty} \frac{\sin 2\pi w \cos \pi w}{w} dw$$

$$\boxed{1}$$

6. (15) Let

$$f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & x \leq 0. \end{cases}$$

Compute the complex Fourier transform of f.

$$\frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{-(1+iw)x}\, dx = \frac{1}{\sqrt{2\pi}} \frac{-1}{1+iw} e^{-(1+iwx)} \Big|_0^{\infty}$$

$$= \frac{1}{\sqrt{2\pi}\,(1+iw)}$$

$$\boxed{\frac{1}{\sqrt{2\pi}\,(1+iw)}}$$