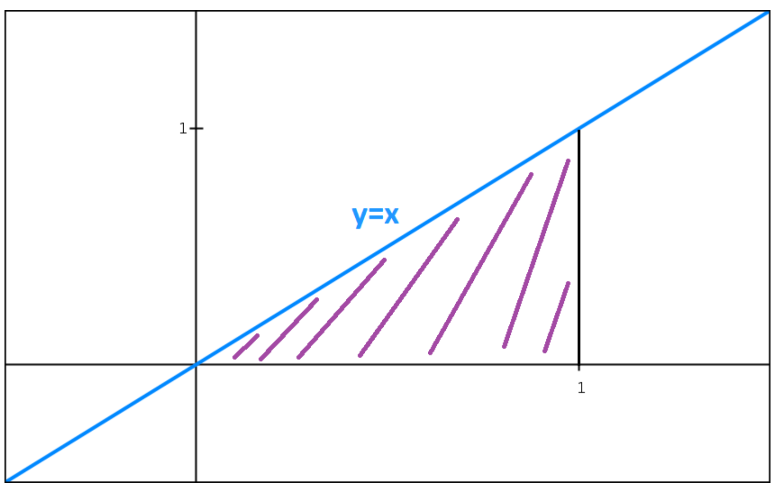
QUIZ 22 SOLUTIONS: LESSONS 29-31
NOVEMBER 17, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Evaluate

$$\int_0^1 \int_y^1 2e^{x^2}\,dx\,dy.$$

<u>Solution</u>: We need to swap the order of integration. We have $0 \le y \le 1$ and $y \le x \le 1$. Sketching this region, we get



Hence, this region is also described by $0 \le x \le 1$ and $0 \le y \le x$. So we may write

$$\begin{aligned}\int_0^1 \int_y^1 2e^{x^2}\,dx\,dy &= \int_0^1 \int_0^x 2e^{x^2}\,dy\,dx \\ &= \int_0^1 2ye^{x^2}\Big|_{y=0}^{y=x}\,dx \\ &= \int_0^1 \left[2xe^{x^2} - 2(0)e^{x^2}\right]\,dx \\ &= \int_0^1 2xe^{x^2}\,dx\end{aligned}$$

By u-substitution, we take $u = x^2$, $du = 2x\,dx$, and $u(0) = (0)^2 = 0$, $u(1) = (1)^2 = 1$. Thus,

$$\begin{aligned}\int_0^1 2xe^{x^2}\,dx &= \int_0^1 e^u\,du \\ &= e^1 - e^0 \\ &= \boxed{e-1}\end{aligned}$$

2. [5 pts] Solve and classify the following system of equations:

$$\begin{cases} x + 2y = 0 \\ 2y + 3z = 8 \\ 2x + 2y + z = 0 \end{cases}$$

Solution: I will solve this by putting the associated augmented matrix into reduced row-echelon form but there are many ways to solve this problem. Moreover, the particular steps that I take to put the augmented matrix into reduced row-echelon form are not the only steps one can take so another person's solution need not match this exactly to be equally correct.

$$\xrightarrow{\text{Translate}} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 2 & 3 & 8 \\ 2 & 2 & 1 & 0 \end{array}\right] \xrightarrow{-2R_1+R_3\to R_3} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 2 & 3 & 8 \\ 0 & -2 & 1 & 0 \end{array}\right] \xrightarrow{2R_2+R_3\to R_3} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 2 & 3 & 8 \\ 0 & 0 & 4 & 8 \end{array}\right]$$

$$\xrightarrow{\frac{1}{4}R_3\to R_3} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 2 & 3 & 8 \\ 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow{-3R_3+R_2\to R_2} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow{-R_2+R_1\to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_2\to R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Therefore, the system is $\boxed{\textbf{consistent independent}}$ and the solution is $\boxed{(x, y, z) = (-2, 1, 2)}$.