

MATH 265 – HANDWRITTEN HOMEWORK 26

Euler's formula: $e^{i\theta} = \cos \theta + i \sin \theta$

1. Express the complex number $z = \frac{1 + 3i}{6 + 8i}$ as $z = re^{i\theta}$ where r is the magnitude (or absolute value) of z , and θ is the argument angle of z in radians.
2. Express $z = 10e^{i\frac{\pi}{6}}$ as $z = a + bi$ where a is the real part of z and b is the imaginary part.
3. Find all values of r such that the complex number $re^{i\frac{\pi}{4}} = a + bi$, where both a and b are integers.
4. Find all real and complex solutions of the equation $z^{10} = 9^{10}$.
5. Find all real and complex solutions to the equation $x^4 - 2x^2 + 1 = 0$.
6. Find all real and complex eigenvalues of the matrix

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 5 & -3 \end{bmatrix}.$$

7. Show that if $p(x)$ is a polynomial with real coefficients and z is a solution of $p(x) = 0$, then $\bar{z}$, the conjugate of z is also a solution of $p(x) = 0$, that is $p(\bar{z}) = 0$.
8. One can identify complex numbers and vectors on the plane through $\mathbb{R}^2$ by $a + bi \equiv \begin{bmatrix} a \\ b \end{bmatrix}$.

Using this identification, find the matrix $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ such that

$$e^{i\phi}(a + bi) \equiv B \begin{bmatrix} a \\ b \end{bmatrix}.$$

Use this to explain geometrically the action of the matrix B on the vector $\begin{bmatrix} a \\ b \end{bmatrix}$.