

Interpretations of Solving Linear Systems

$$2 \times 2: \text{ Solve: } \begin{cases} 2x - y = 4 & \text{--- (1)} \\ x + 3y = 9 & \text{--- (2)} \end{cases}$$

Solve by elimination & substitution:

$$\textcircled{1} \Rightarrow y = 2x - 4$$

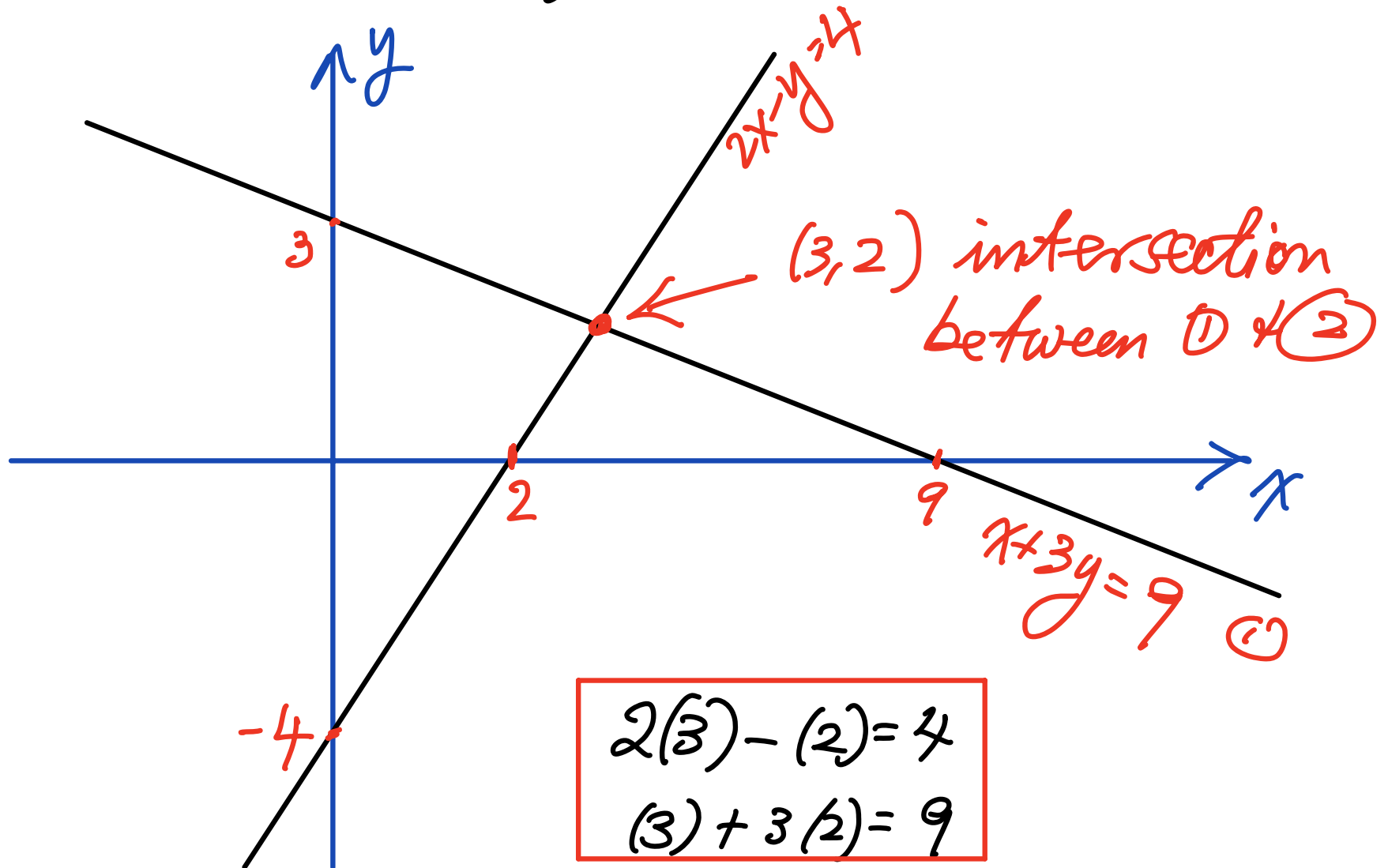
$$\textcircled{2} \Rightarrow x + 3(2x - 4) = 9$$

$$7x = 9 + 12 = 21$$

$$\text{Hence } x = 3, y = 2(3) - 4 = 2$$

$$\text{Solution: } (x, y) = (3, 2), \text{ or } \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

(I) Finding intersection between 2 st. lines
(Look at each equation or row)



(II) Writing vector as Linear Combination (look at "columns")

vector
addition

$$\begin{cases} 2x - y = 4 \\ x + 3y = 9 \end{cases}$$

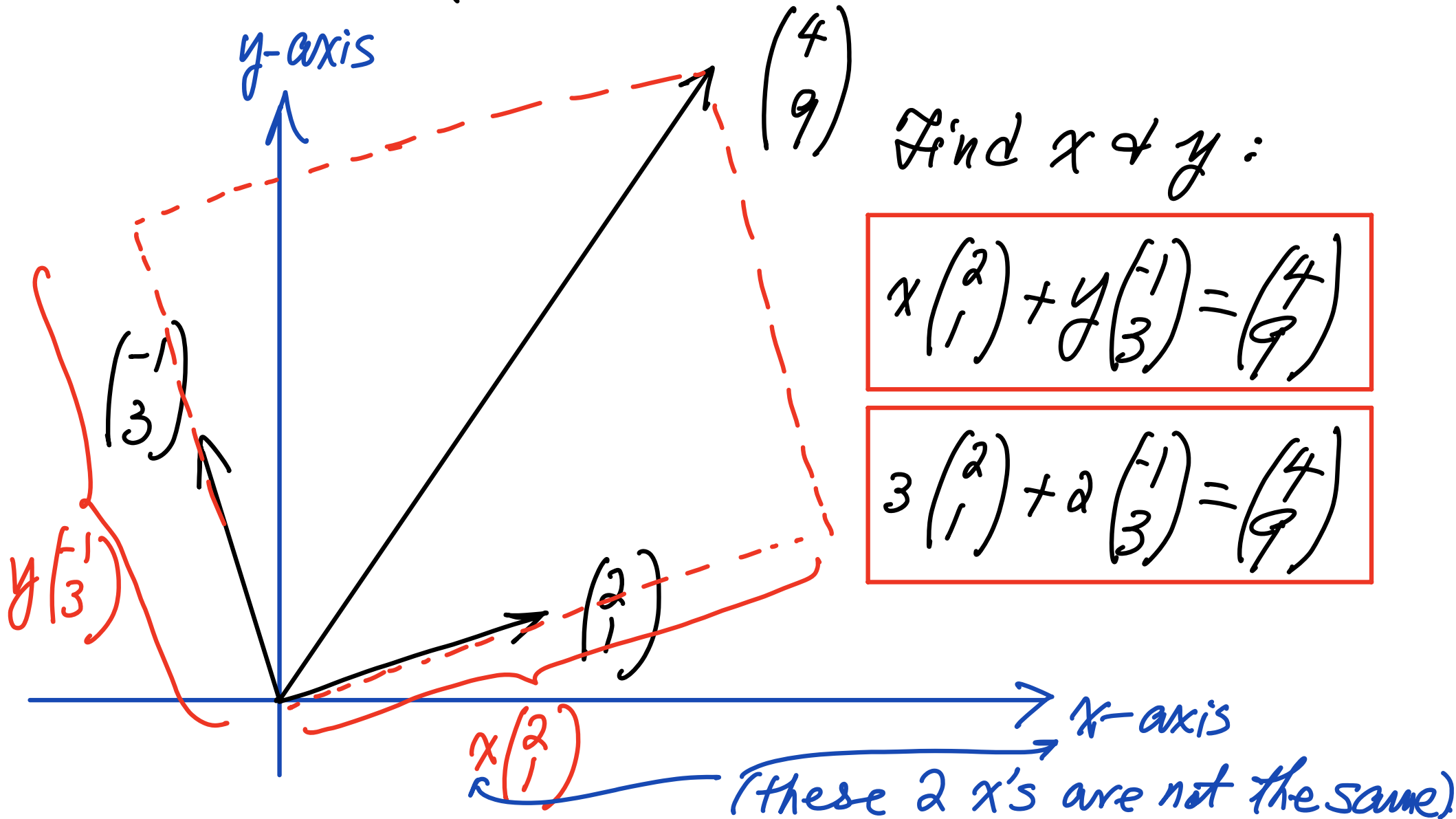
$$\begin{pmatrix} 2x \\ x \end{pmatrix} + \begin{pmatrix} -y \\ 3y \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \end{pmatrix}$$

scalar
multipli-
cation

$$x \begin{pmatrix} 2 \\ 1 \end{pmatrix} + y \begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \end{pmatrix}$$

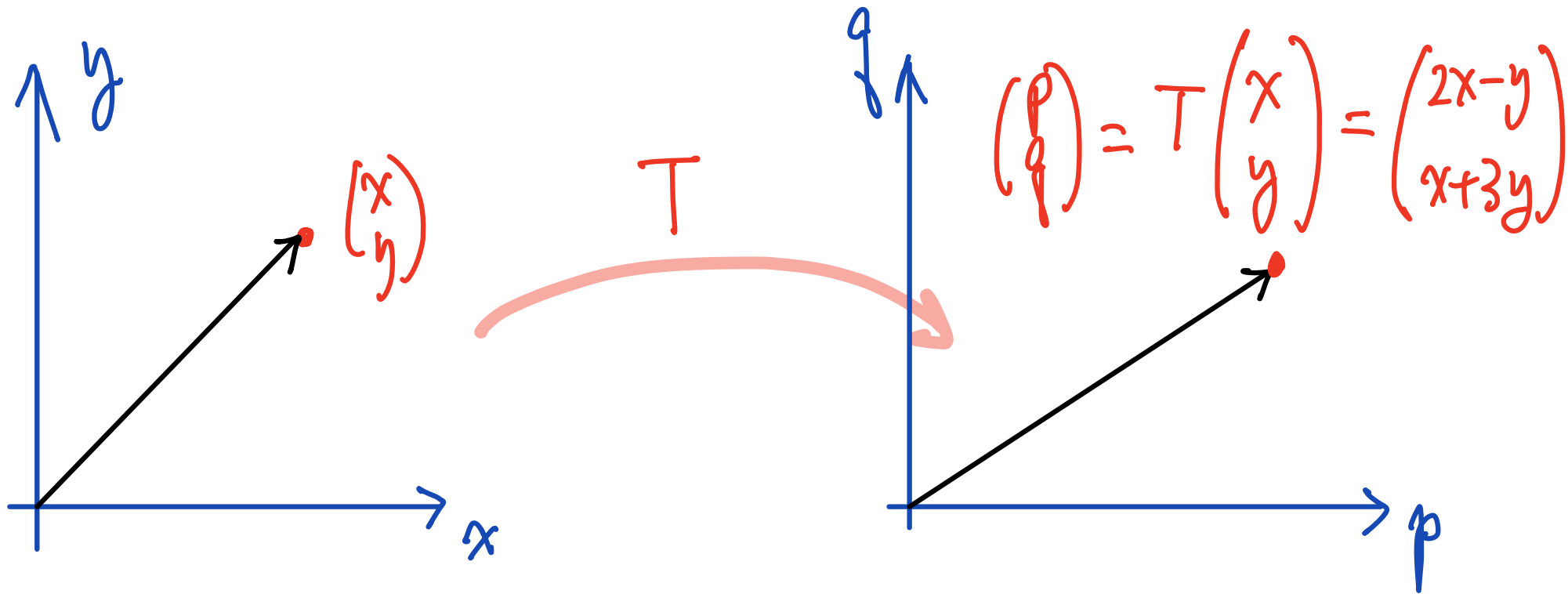
unknowns

(II) Writing vector as Linear Combination
(look at "columns")



(II) Linear Transformation of Vectors

$$\begin{pmatrix} x \\ y \end{pmatrix} \xrightarrow{T} T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x - y \\ x + 3y \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix}$$



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Solving $\begin{cases} 2x-y = 4 \\ x+3y = 9 \end{cases}$ is equivalent

to finding $\begin{pmatrix} x \\ y \end{pmatrix}$ such that $T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \end{pmatrix}$

(ie. $\begin{pmatrix} 4 \\ 9 \end{pmatrix}$ is the image of $\begin{pmatrix} x \\ y \end{pmatrix}$ under T .)

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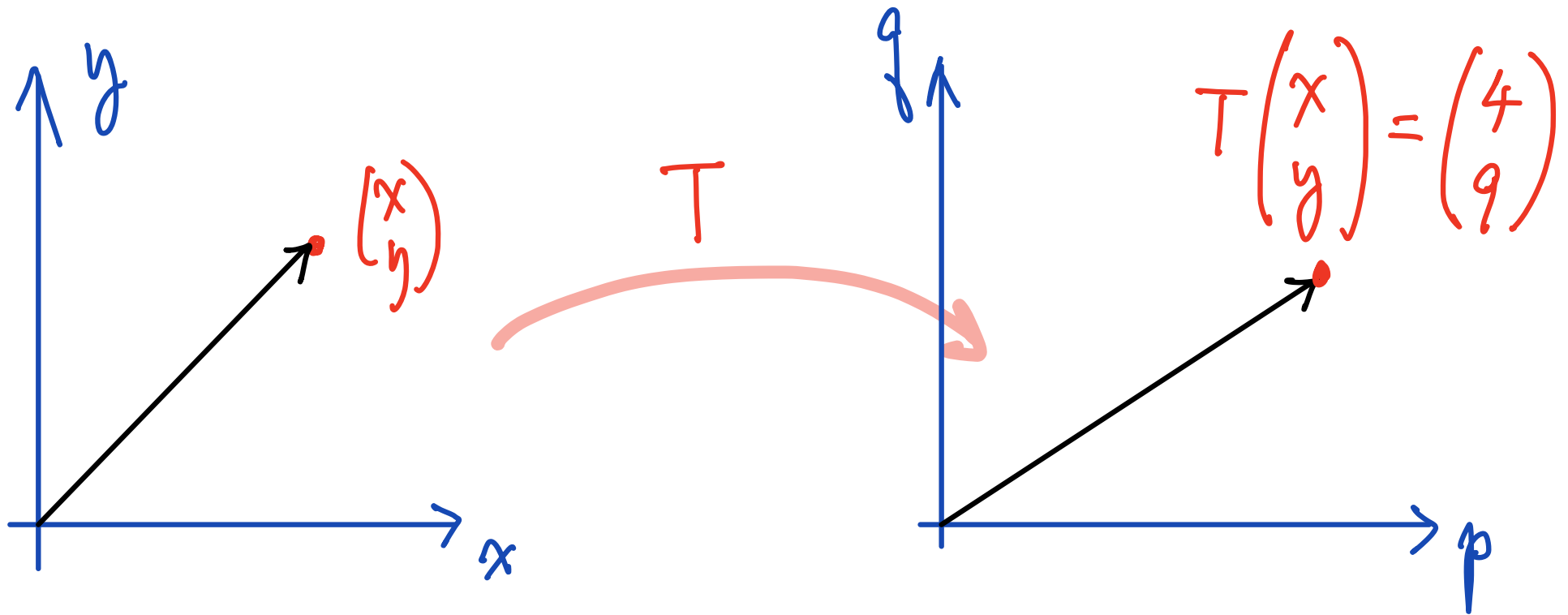
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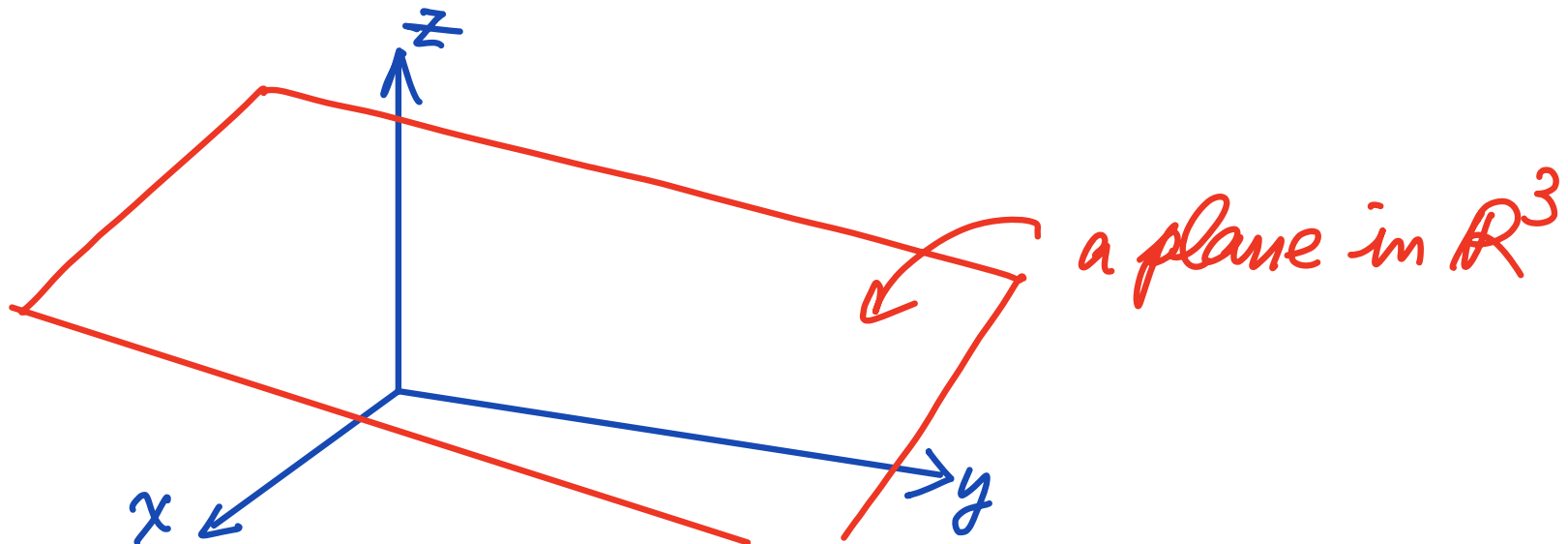


Example in $\mathbb{R}^3$

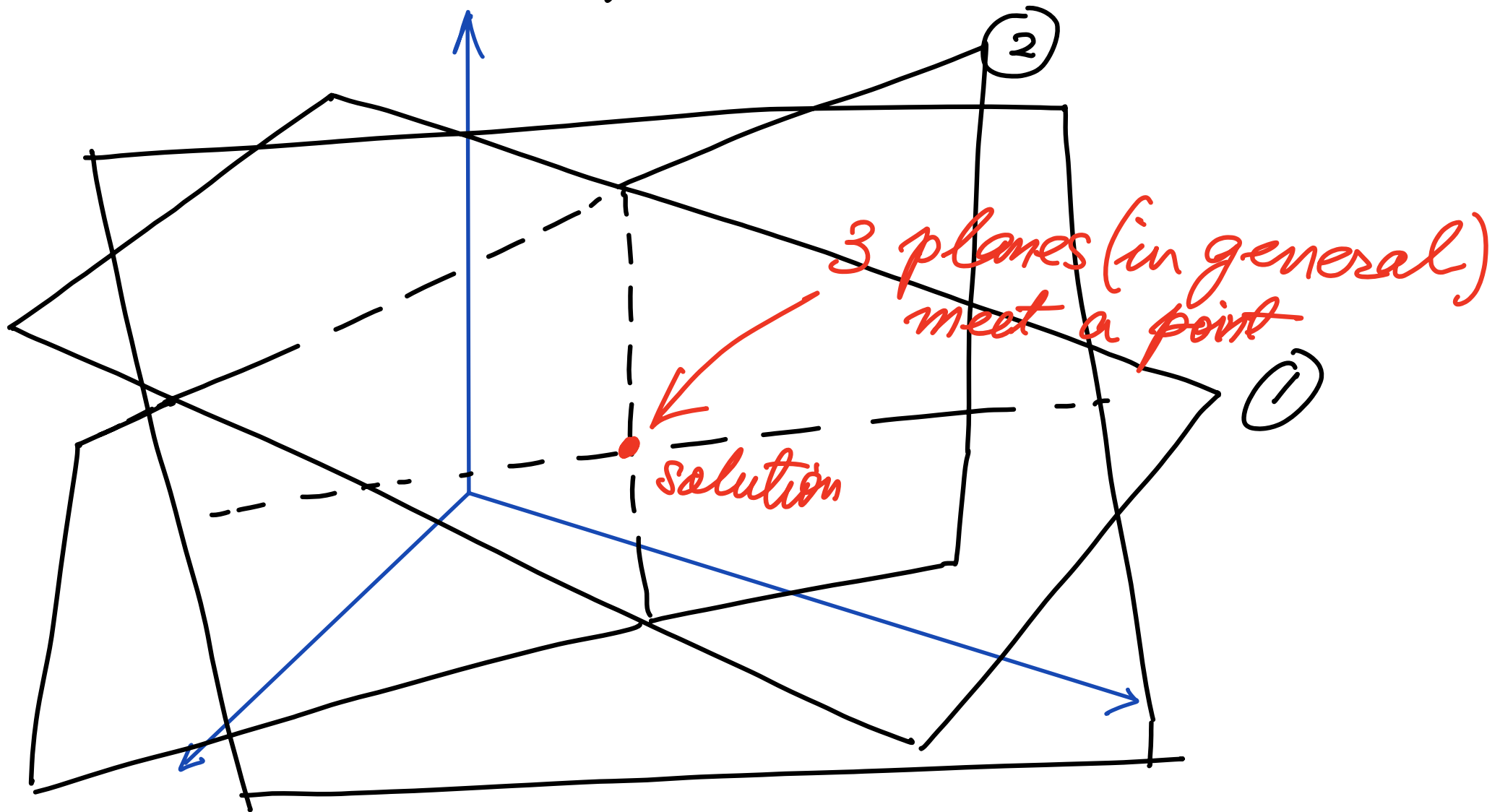
3x3 system:

$$(*) \begin{cases} x + 2y + z = 1 & \text{--- ①} \\ 3x + y + 4z = 0 & \text{--- ②} \\ 2x + 2y + 3z = 2 & \text{--- ③} \end{cases}$$

Each equation represents a plane in $\mathbb{R}^3$



(I) Solving (*) is to find the intersection
between 3 planes in $\mathbb{R}^3$.



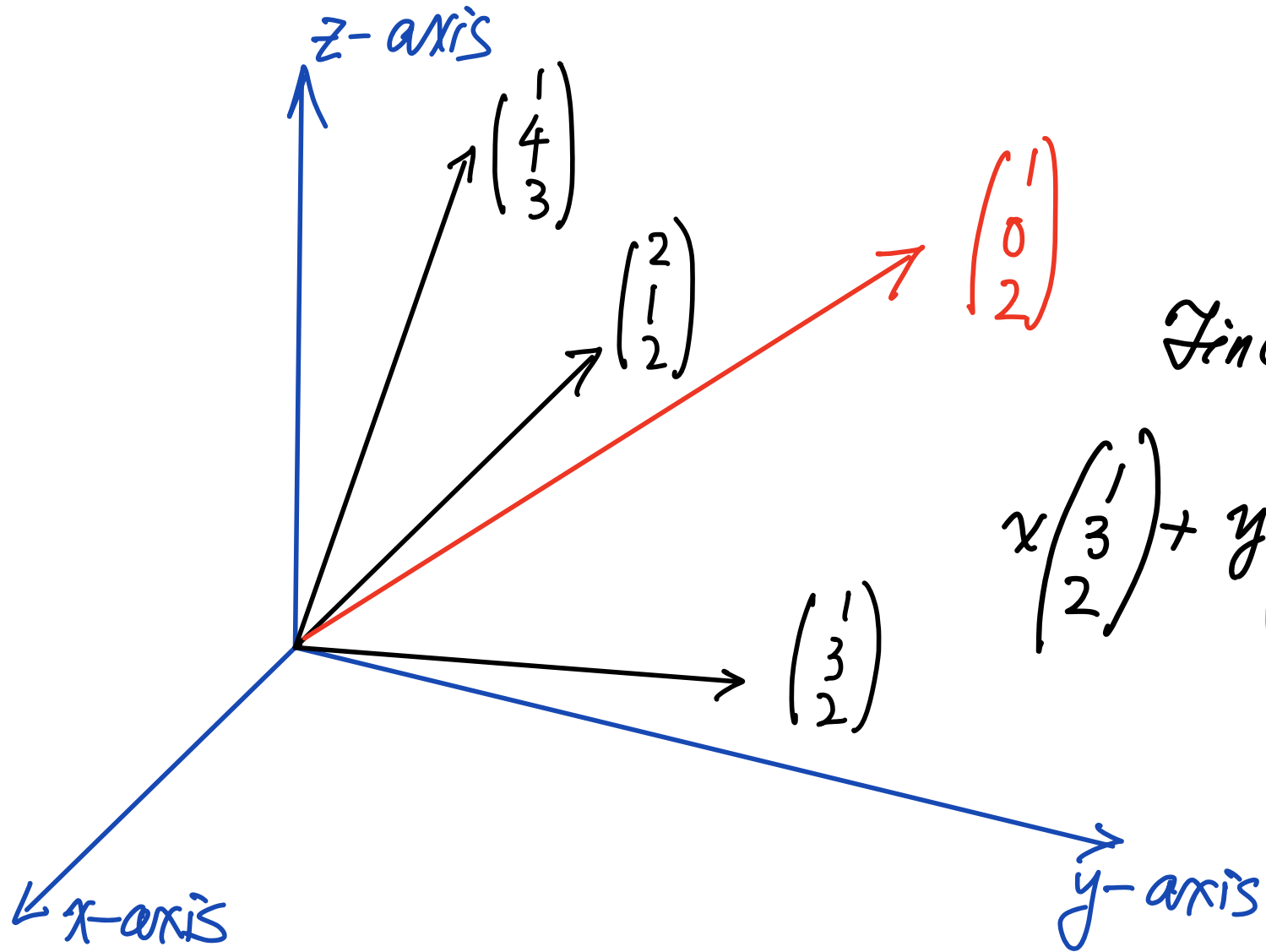
(II) Writing Vector as linear combination

$$\begin{cases} x + 2y + z = 1 \\ 3x + y + 4z = 0 \\ 2x + 2y + 3z = 2 \end{cases}$$

$$\begin{pmatrix} x \\ 3x \\ 2x \end{pmatrix} + \begin{pmatrix} 2y \\ y \\ 2y \end{pmatrix} + \begin{pmatrix} z \\ 4z \\ 3z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$$x \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + y \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} + z \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

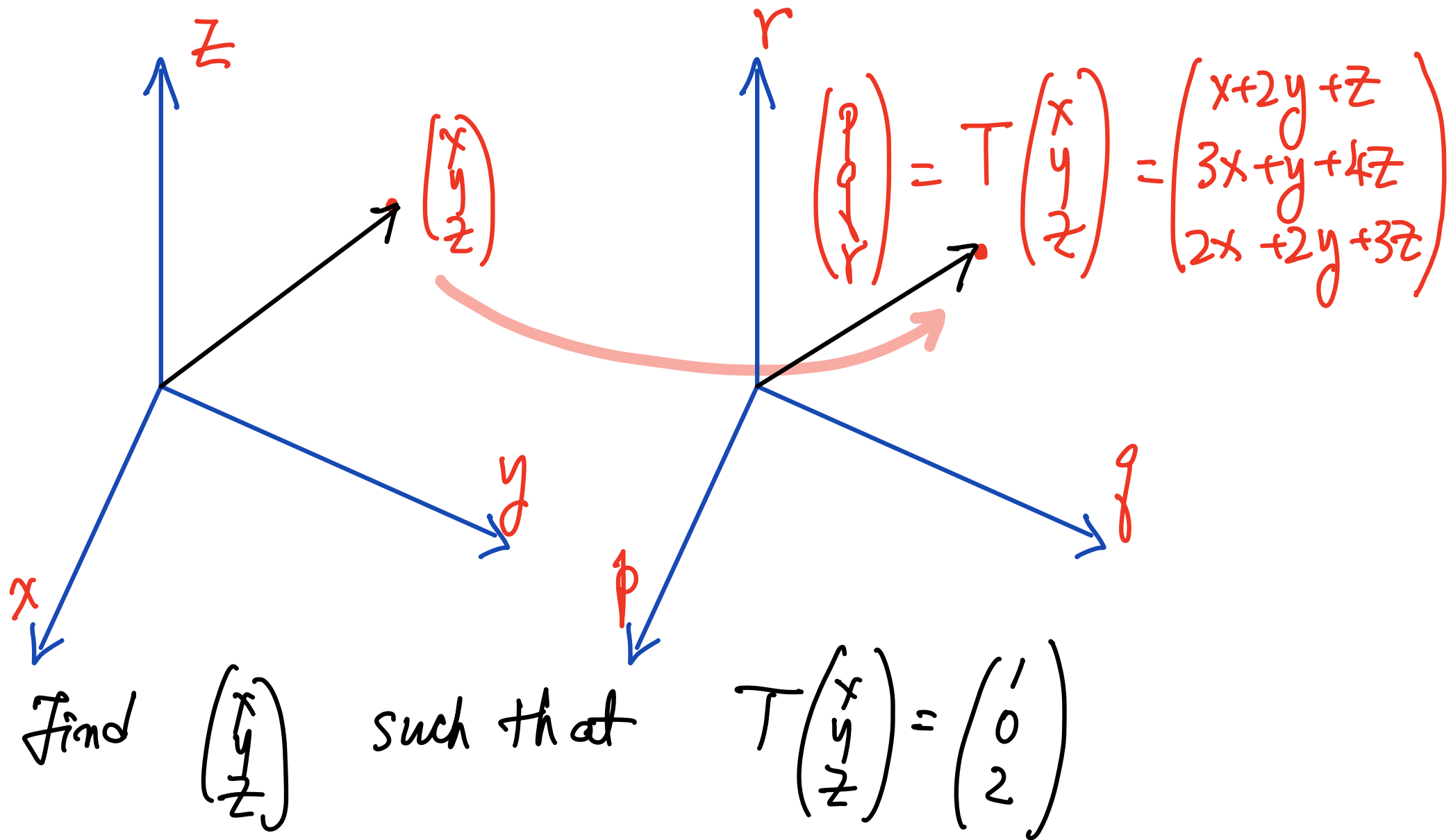
(II) Writing Vector as linear combination



Find x, y, z :

$$x \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + y \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} + z \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

(III) Linear Transformation



Solution of (*) (Using elimination & substitution)

$$\begin{cases} x + 2y + z = 1 & \text{--- ①} \\ 3x + y + 4z = 0 & \text{--- ②} \\ 2x + 2y + 3z = 2 & \text{--- ③} \end{cases}$$

$$\text{①} \Rightarrow x = 1 - 2y - z$$

$$\text{②} \Rightarrow 3(1 - 2y - z) + y + 4z = 0 \Rightarrow -5y + z = -3$$

$$\text{③} \Rightarrow 2(1 - 2y - z) + 2y + 3z = 2 \Rightarrow \underbrace{-2y + z = 0}$$

$$\text{Solution: } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$$

$$x = 1 - 2(1) - 2 = -3$$

$$\begin{aligned} y &= 1 \\ z &= 2 \end{aligned}$$

"Just to make sure":

$$(I): \begin{cases} (-3) + 2(1) + (2) = 1 \\ 3(-3) + (1) + 4(2) = 0 \\ 2(-3) + 2(1) + 3(2) = 2 \end{cases}$$

$$(II): -3 \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + 1 \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$