MA 351 Fall 2023 Quiz 1 (Yip)

Consider the following 2x2 linear system:

$$\begin{cases} 2x - y = 1 \\ x + y = 5 \end{cases}$$

(I) (i) Solve the above system.

(ii) Plot the above system as two straight lines in xy-plane and clearly indicate the solution as an intersecting point.

(II) (i) Write the above system as a linear combination of vectors:

$$x\vec{u} + y\vec{v} = \vec{w}$$

Identify $\vec{u}$, $\vec{v}$ and $\vec{w}$.

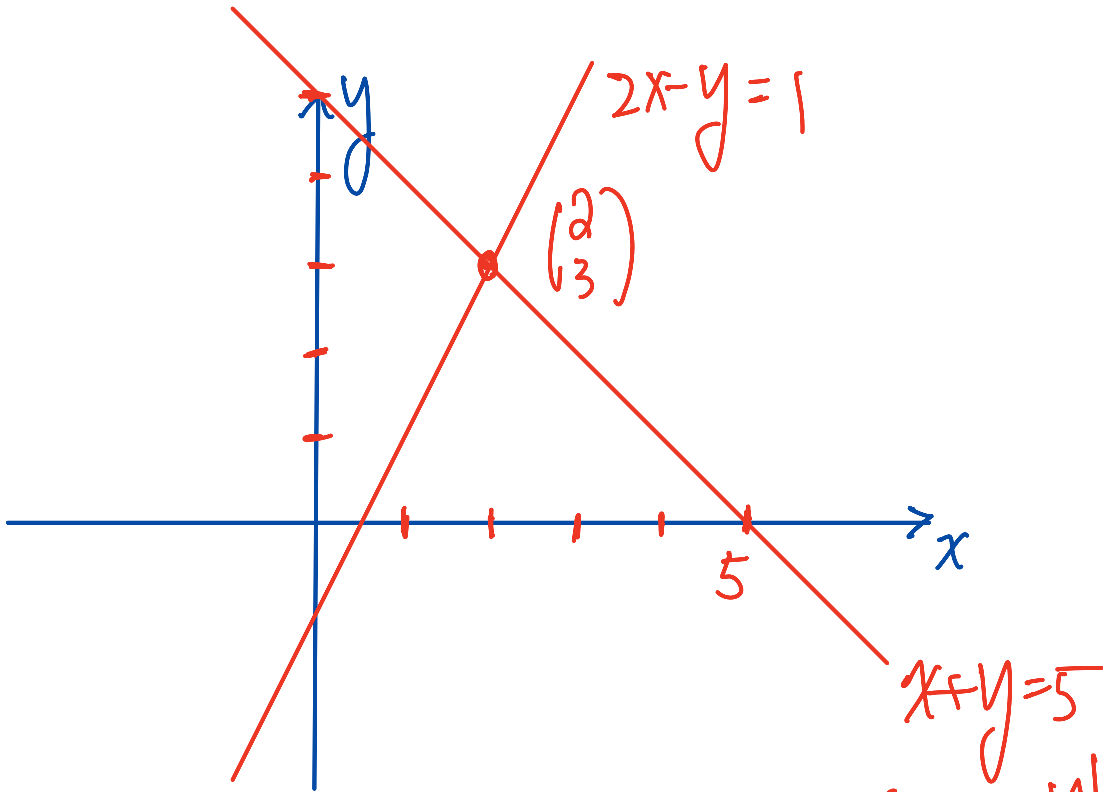
(ii) Illustrate/plot the linear combination using vector addition

(You can utilize the grid in the blank pages.)

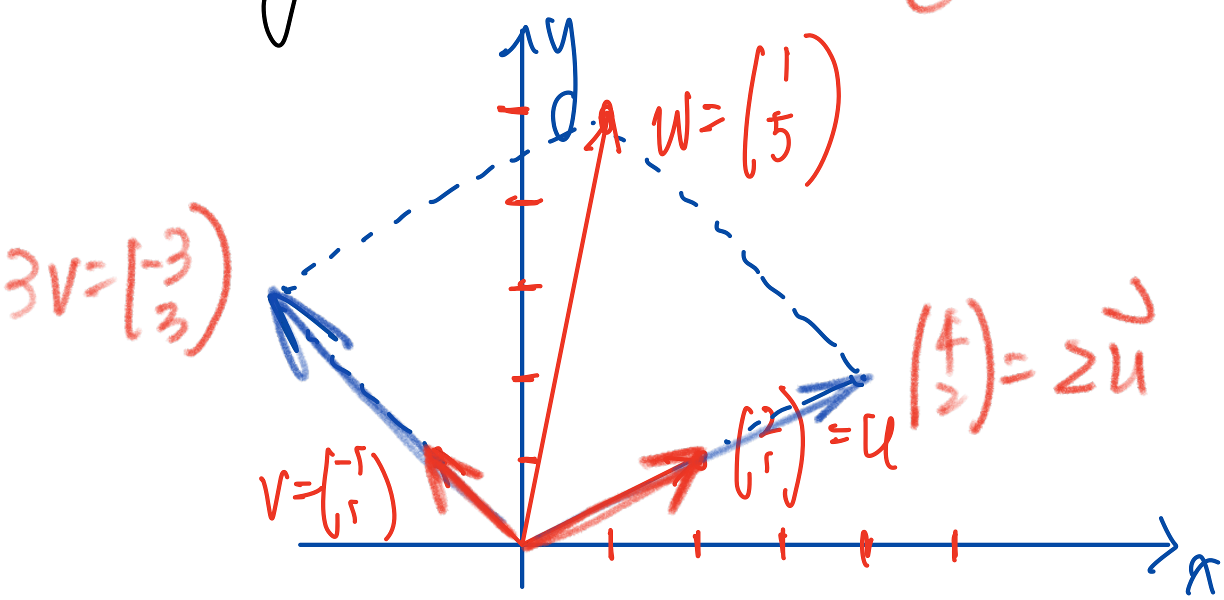
Solution (I) i) $\begin{cases} 2x - y = 1 \\ x + y = 5 \end{cases}$

$+ \Longrightarrow \quad x = 2, \; y = 3$

(ii)



(II) $\begin{cases} 2x - y = 1 \\ x + y = 5 \end{cases} \iff \underset{2}{\cancel{x}}\overbrace{\begin{pmatrix}2\\1\end{pmatrix}}^{u} + \underset{3}{\cancel{y}}\overbrace{\begin{pmatrix}-1\\1\end{pmatrix}}^{v} = \overbrace{\begin{pmatrix}1\\5\end{pmatrix}}^{w}$



Quiz #2 Consider $\begin{cases} 7x - y = \lambda x \\ -6x + 8y = \lambda y \end{cases}$

Solve the system for $\lambda = 5, 10, 15$

$\lambda = 5$: $\begin{cases} 2x - y = 0 \\ -6x + 3y = 0 \end{cases} \Rightarrow \left(\begin{array}{cc|c} 2 & -1 & 0 \\ -6 & 3 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 2 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$

$y = s, \ x = \frac{s}{2}$

Solution : $\begin{pmatrix} x \\ y \end{pmatrix} = s \begin{pmatrix} 1/2 \\ 1 \end{pmatrix}$ (infinitely many solution)

$\lambda = 10$ $\begin{cases} -3x - y = 0 \\ -6x - 2y = 0 \end{cases}$ $\left(\begin{array}{cc|c} -3 & -1 & 0 \\ -6 & -2 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} -3 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$

$y = s, \quad x = -\frac{1}{3} s$

Solution : $\begin{pmatrix} x \\ y \end{pmatrix} = s \begin{pmatrix} -1/3 \\ 1 \end{pmatrix}$ (inf. many solutions)

$\lambda = 15$ $\begin{cases} -8x - y = 0 \\ -6x - 7y = 0 \end{cases}$ $\left(\begin{array}{cc|c} -8 & -1 & 0 \\ -6 & -7 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 8 & 1 & 0 \\ 6 & 7 & 0 \end{array}\right)$

$\to \left(\begin{array}{cc|c} 1 & 1/8 & 0 \\ 6 & 7 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & \frac{1}{8} & 0 \\ 0 & \frac{50}{8} & 0 \end{array}\right)$

Solution: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ (only the trivial soln) $x = 0, \ y = 0$

Quiz 3

$$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \boxed{T} \longrightarrow \begin{pmatrix} p \\ q \end{pmatrix}$$

$$\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax+by \\ cx+dy \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \longrightarrow \begin{pmatrix} -4 \\ 5 \end{pmatrix}, \qquad \begin{pmatrix} 1 \\ 2 \end{pmatrix} \longrightarrow \begin{pmatrix} -9 \\ 8 \end{pmatrix}$$

$T = ?$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ 5 \end{pmatrix} \iff \begin{aligned} a+b &= -4 \\ c+d &= 5 \end{aligned}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -9 \\ 8 \end{pmatrix} \iff \begin{aligned} a+2b &= -9 \\ c+2d &= 8 \end{aligned}$$

$b = -5$
$a = 1$

$d = 3$
$c = 2$

$$T = \begin{pmatrix} 1 & -5 \\ 2 & 3 \end{pmatrix}$$

Quiz 4

$\alpha = ?$ such that

(i) unique soln

(ii) infinitely many soln

(iii) no soln.

1.1.2.

$$\begin{cases} 2x + 3y = 7 \\ 6x + 9y = \alpha \end{cases}$$

1.1.3.

$$\begin{cases} 2x + 3y = 7 \\ 3x + \alpha y = 10 \end{cases}$$

1.1.4.

$$\begin{cases} 2x + 3y = 0 \\ 3x + \alpha y = 0 \end{cases}$$

1.1.5.

$$\begin{cases} 2x + \alpha y = 7 \\ \alpha x + 8y = 14 \end{cases}$$

1.1.2

$$\left(\begin{array}{cc|c} 2 & 3 & 7 \\ 6 & 9 & \alpha \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 2 & 3 & 7 \\ 0 & 0 & \alpha-21 \end{array}\right)$$

(free variable)

(i) is not possible

(ii) $\alpha - 21 = 0 \iff \alpha = 21$

(iii) $\alpha - 21 \neq 0 \iff \alpha \neq 21$

1.1.3

$$\left(\begin{array}{cc|c} 2 & 3 & 7 \\ 3 & \alpha & 10 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & 3/2 & 7/2 \\ 3 & \alpha & 10 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{cc|c} 1 & 3/2 & 7/2 \\ 0 & \alpha - 9/2 & -\frac{1}{2} \end{array}\right)$$

$10 - \frac{21}{2}$

(i) $\alpha \neq 9/2$

(ii) not possible

(iii) $\alpha = 9/2$

1.1.4 $\left(\begin{array}{cc|c} 2 & 3 & 0 \\ 3 & \alpha & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 3/2 & 0 \\ 3 & \alpha & 0 \end{array}\right)$

$\to \left(\begin{array}{cc|c} 1 & 3/2 & 0 \\ 0 & \alpha - 9/2 & 0 \end{array}\right)$

(i) $\alpha \neq 9/2$

(ii) $\alpha = 9/2$

(iii) Not possible

($AX = 0$ always has a solution, $X = 0$)

1.1.5 $\left(\begin{array}{cc|c} 2 & \alpha & 7 \\ \alpha & 8 & 14 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & \alpha/2 & 7/2 \\ \alpha & 8 & 14 \end{array}\right)$

$\to \left(\begin{array}{cc|c} 1 & \alpha/2 & 7/2 \\ 0 & 8 - \frac{\alpha^2}{2} & 14 - \frac{7\alpha}{2} \end{array}\right) \to \left(\begin{array}{cc|c} 1 & \frac{\alpha}{2} & 7/2 \\ 0 & 16 - \alpha^2 & 28 - 7\alpha \end{array}\right)$

(i) $16 - \alpha^2 \neq 0 \iff \alpha \neq 4, -4$

(ii) $16 - \alpha^2 = 0$ and $28 - 7\alpha = 0$
$\alpha = 4, -4$ and $\alpha = 7$
$\Big\}$ $\alpha = 4$

(iii) $16 - \alpha^2 = 0$ and $28 - 7\alpha \neq 0$
$\alpha = 4, -4,$ $\alpha \neq 4$
$\Big\}$ $\alpha = -4$