

Quiz 11

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

(1) Find B s.t. $AB = I$

(2) Find C s.t. $CA = I$

$$(i) \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}^{2 \times 3} \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}^{3 \times 2} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^{2 \times 2}$$

$$a + 2c + e = 1$$

$$a + c + e = 0$$

$$\begin{pmatrix} 1 & 2 & 1 & | & 1 \\ 1 & 1 & 1 & | & 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 2 & 1 & | & 1 \\ 0 & -1 & 0 & | & -1 \end{pmatrix}$$

$$\Rightarrow c = 1, \quad a + 2c + e = 1 \\ a + e = -1$$

$$a = -1 - e$$

$$c = 1$$

$$e = e \text{ (free)}$$

$$b + 2d + f = 0$$

$$b + d + f = 1$$

$$\begin{pmatrix} 1 & 2 & 1 & | & 0 \\ 1 & 1 & 1 & | & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 1 & | & 0 \\ 0 & -1 & 0 & | & 1 \end{pmatrix}$$

$$\Rightarrow d = -1 \quad b + 2d + f = 0 \\ b + f = 2$$

$$b = 2 - f$$

$$d = -1$$

$$f = f \text{ (free)}$$

$$\begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1-e & 2-f \\ 1 & -1 \\ e & f \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$(2) \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}^{3 \times 2} \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}^{2 \times 3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}^{3 \times 3}$$

$$\begin{aligned} a+b &= 1, & 2a+b &= 0 & a+b &= 0 \\ c+d &= 0 & 2c+d &= 1 & c+d &= 0 \\ e+f &= 0 & 2e+f &= 0 & e+f &= 1 \end{aligned}$$

Contradiction $\Rightarrow$ not possible

Quiz 12

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 1 & -1 \end{pmatrix}$$

(1) Find B s.t. $AB=I$

(2) Find C s.t. $CA=I$

$$(1) \begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 1 & -1 \end{pmatrix}^{3 \times 2} \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}^{2 \times 3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}^{3 \times 3}$$

$$a = 1$$

$$b = 0$$

$$c = 0$$

$$2a+d = 0$$

$$2b+e = 1$$

$$2c+f = 0$$

$$a-d = 0$$

$$b-e = 0$$

$$c-f = 0$$

$$\begin{aligned} d &= -2a = -2 \\ d &= a = 1 \end{aligned}$$

!!

not possible

$$(2) \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}^{2 \times 3} \begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 1 & -1 \end{pmatrix}^{3 \times 2} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^{2 \times 2}$$

$$a + 2b + c = 1, \quad b - c = 0$$

$$d + 2e + f = 0$$

$$e - f = 1$$

$$a + 2b + c = 1$$

$$b - c = 0$$

↓
free

$$b = c$$

$$a = 1 - 2b - c$$

$$= 1 - 3c$$

$$d + 2e + f = 0$$

$$e - f = 1$$

↓
free

$$e = 1 + f$$

$$d = -2e - f$$

$$= -2 - 3f$$

$$\begin{pmatrix} 1-3c & c & c \\ -2-3f & 1+f & f \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$