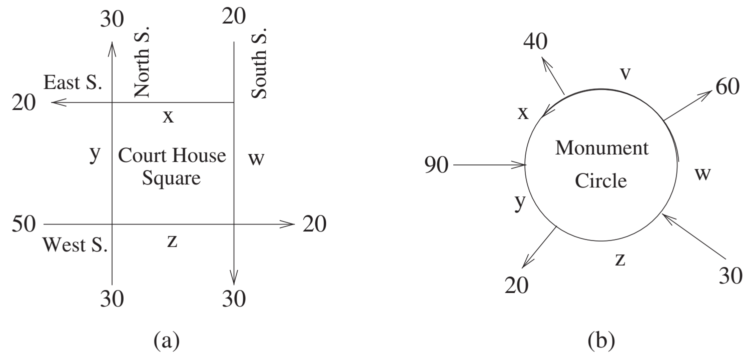

FIGURE 1.24 Two traffic patterns.

The final output to you should be a simple Yes or No. You do not want to see matrices!

(c) One of your assistant engineers comments that it would be easier for the technician in part (b) to use MATLAB's rank command rather than rref. What does your assistant have in mind?

1.3.2 Applications to Traffic Flow

An interesting context in which linearly dependent systems of equations can arise is in the study of traffic flow. Figure 1.24a is a map of the downtown area of a city. Each street is one-way in its respective direction. The numbers represent the average number of cars per minute that enter or leave a given street at 3:30 p.m. The variables also represent average numbers of cars per minute. Of course, barring accidents, the total number of cars entering any intersection must equal the total number leaving. Thus, from the intersection of East and North, we see that $x + y = 50$. Continuing counterclockwise around the square, we get the system

$$\begin{aligned} x + y \qquad\qquad &= 50 \\ y + z \qquad &= 80 \\ z + w &= 50 \\ x \qquad\qquad + w &= 20 \end{aligned} \tag{1.37}$$

Notice also that in Figure 1.24, the total number of cars entering the street system per minute is $20 + 30 + 50 = 100$ while the total number leaving is $20 + 30 + 20 + 30 = 100$. All of our examples share the property that the total number of cars per minute that enter the street system per minute equals the total number that leave the street system per minute. *We assume that the total number of cars per minute on the street system at any given time equals the total number of cars entering the street system per minute which equals the total number of cars leaving the street system per minute.* Thus we augment system 1.37 with the additional equation

$$x + y + z + w = 100$$

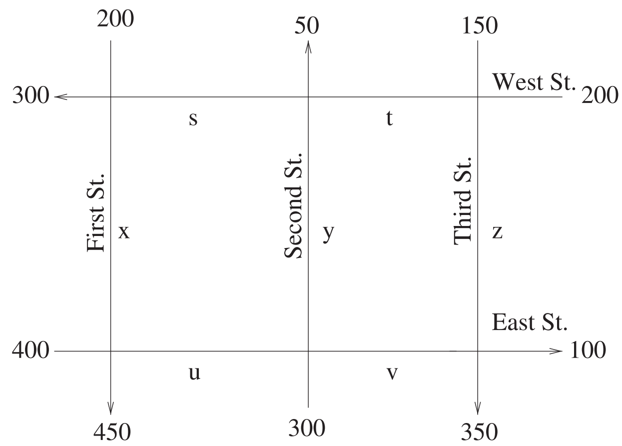

FIGURE 1.25 Exercise 3.

In this case, this equation is dependent on the other equations; it equals half their sum. In other cases, however, it can add additional information.

In the exercises that follow, you will analyze this system as well as the system that describes figure (b). The type of analysis done in these exercises can be applied in any context where some quantity is flowing through prescribed channels. In circuit theory, the statement that the amount of current entering a node equals the amount leaving is called Kirchhoff's current law.

Self-Study Questions

1.7 ✓In Figure 1.25 write the linear equation that describes the traffic flow at the intersection of West and Third Streets.

1.8 ✓In Figure 1.25, write the linear equation that describes the traffic flow at the intersection of East and Second Streets.

1.9 ✓In Figure 1.24b, write the linear equation that describes the traffic flow at the intersection of Monument Circle with the street labeled 90.

EXERCISES

1.90 Find all solutions to the system that describes Figure 1.24a. Use w as your free variable. Then answer the following:

(a) ✓Suppose that over the month of December the traffic on South Street in front of the courthouse at 3:30 p.m. ranged from six to eight cars per minute. Determine which of the streets on Courthouse Square had the greatest volume of traffic. What were the maximum and minimum levels of traffic flow on this street?

(b) Suppose that it is observed that in June the traffic flow past the courthouse is heaviest on West Street. Prove that $0 \leq w < 10$.

(c) Sleezie's Construction wants to close down West Street for six months. The City Council refused to grant the permit. Why? Explain on the basis of the solution to the system of Figure 1.24a.

1.91 Find all solutions to the system that describes Figure 1.24b. Use v as your free variable.

1.92 Figure 1.25 shows the traffic flow in a town at 3:30 p.m.

(a) Find the system of equations that describes the traffic flow?

(b) The section of Third Street between East and West is under construction. Assuming that as few cars as possible use this block, what are the possible ranges of traffic flow on each of the other blocks? [*Hint:* Note that none of the variables can take on negative values.]

1.4 COLUMN SPACE AND NULLSPACE

The two most fundamental questions concerning any system of equations are: (a) Is it solvable? (b) If it is solvable, what is the nature of the solution set? In this section we begin the discussion of both issues. This discussion will be completed in Section 2.3. Example 1.15 addresses the first question.

EXAMPLE 1.15

Describe geometrically the set of vectors $B = [b_1, b_2, b_3]^t$ for which the system below is solvable.

$$\begin{aligned} x \qquad + 2z &= b_1 \\ x + y + 3z &= b_2 \\ y + z &= b_3 \end{aligned}$$

Solution. The system may be written as a single matrix equation:

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} x \qquad + 2z \\ x + y + 3z \\ \qquad y + z \end{bmatrix} = x \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + y \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + z \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \tag{1.38}$$

We denote the three columns on the right by A_1, A_2, and A_3, respectively, so that our system becomes

$$B = xA_1 + yA_2 + zA_3$$

We conclude that our system is solvable if and only if B is in the span of the A_i.