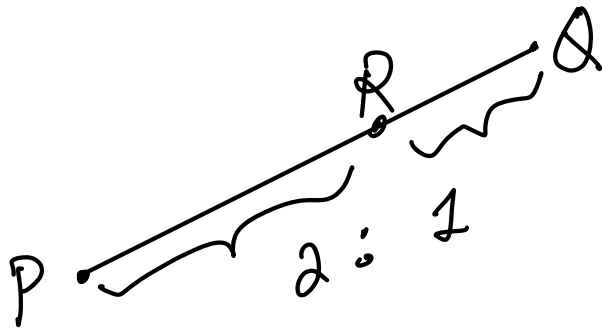


Quiz 1 Solution

Let P and Q be two points in space and PQ be the line segment connecting P and Q . Find the point R on PQ such that the ratio between the length PR and RQ is $2:1$.



$$\vec{PR} = 2\vec{RQ}$$

$$R - P = 2(Q - R)$$

$$R - P = 2Q - 2R$$

$$3R = P + 2Q$$

$$R = \frac{P + 2Q}{3}$$

Quiz 2 Consider the following system of linear eq:

$$x + y + z = 2$$

$$x + 2y + z = 3$$

$$x + y + (c^2 - 3)z = c$$

c - a parameter

Discuss the solvability in terms of c .

When does the system have (i) unique solution, (ii) infinitely many solution; (iii) no solution.

Solve the system when solution exists

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & c^2 - 3 & c \end{array} \right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & c^2 - 4 & c - 2 \end{array} \right)$$

When $c^2 - 4 \neq 0$ i.e. $c \neq 2, -2$, there is unique solution.

$$z = \frac{c-2}{c^2-4} = \frac{1}{c+2}, \quad y = 1$$

$$x = 2 - \frac{1}{c+2} - 1 = \frac{c+1}{c+2}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{c+1}{c+2} \\ 1 \\ \frac{1}{c+2} \end{pmatrix}$$

When $c^2 - 4 = 0$ and $c - 2 = 0$, i.e. $c = 2$,
then infinitely many solution.

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$z = \alpha \text{ (free)}, \quad y = 1, \quad x = 2 - \alpha - 1 = 1 - \alpha$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 - \alpha \\ 1 \\ \alpha \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

When $c^2 - 4 = 0$, $c - 2 \neq 0$, i.e. $c = -2$
no solution

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & -4 \end{array} \right)$$

Quiz 3

$$p_1(x) = 1 + 2x + 3x^2$$

$$p_2(x) = x - x^2$$

$$p_3(x) = 2 + 3x + 7x^2$$

Determine $\text{Span}\{p_1, p_2, p_3\}$, i.e. find ALL $p(x) = a + bx + cx^2$ such that $p(x)$ is from $\text{Span}\{p_1, p_2, p_3\}$

$$\underline{a} + \underline{b}x + \underline{c}x^2 = \underline{c_1}(1 + 2x + 3x^2) + \underline{c_2}(x - x^2) + \underline{c_3}(2 + 3x + 7x^2)$$

$$\begin{aligned} c_1 + 2c_3 &= a \\ 2c_1 + c_2 + 3c_3 &= b \\ 3c_1 - c_2 + 7c_3 &= c \end{aligned} \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 2 & a \\ 2 & 1 & 3 & b \\ 3 & -1 & 7 & c \end{array} \right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 2 & a \\ 0 & 1 & -1 & b-2a \\ 0 & -1 & 1 & c-3a \end{array} \right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 2 & a \\ 0 & 1 & -1 & b-2a \\ 0 & 0 & 0 & b+c-5a \end{array} \right)$$



Need $b+c-5a=0$

Quiz 4 Let $A = \begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 2 & -4 & 4 \\ 1 & 3 & 0 & 4 \end{pmatrix}$

Write $\text{Col}(A)$ and $\text{Null}(A)$ as spans of lin. ind. vectors. Make sure you show that your lists are lin. ind.

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 0 \\ 2 & 2 & -4 & 4 & 0 \\ 1 & 3 & 0 & 4 & 0 \end{array} \right)$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 0 \\ 0 & -2 & -2 & -2 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{array} \right)$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

free variables
the same!

$$\text{Col}(A) = \text{Span} \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \right\}$$

lin ind. because there are no free var.

$$\left(\begin{array}{cc|c} 1 & 2 & 0 \\ 2 & 2 & 0 \\ 1 & 3 & 0 \end{array} \right) \rightarrow \left(\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & -2 & 0 \\ 0 & 1 & 0 \end{array} \right) \rightarrow \left(\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right)$$

$$\text{Null}(A) = \{X : AX = 0\}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 0 \\ 2 & 2 & -4 & 4 & 0 \\ 1 & 3 & 0 & 4 & 0 \end{array} \right) \rightarrow \left(\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ 1 & 2 & -1 & 3 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$x_3 = \alpha, \quad x_4 = \beta \quad (\text{free})$$

$$x_2 = -\alpha - \beta$$

$$\begin{aligned} x_1 &= -2x_2 + x_3 - 3x_4 \\ &= 2\alpha + 2\beta + \alpha - 3\beta \\ &= 3\alpha - \beta \end{aligned}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 3\alpha - \beta \\ -\alpha - \beta \\ \alpha \\ \beta \end{pmatrix} = \alpha \begin{pmatrix} 3 \\ -1 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\text{Null}(A) = \text{Span} \left\{ \begin{pmatrix} 3 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 0 \\ 1 \end{pmatrix} \right\} \quad \text{lin ind.}$$

$$c_1 \begin{pmatrix} 3 \\ -1 \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} -1 \\ -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{array}{l} \leftarrow c_1 = 0 \\ \leftarrow c_2 = 0 \end{array}$$