Quiz 5 Solution

(1) $$M = \left\{ \begin{pmatrix} a+c+2d & b+c+d \\ -b-c-d & -a-c-2d \end{pmatrix} : a,b,c,d \in \mathbb{R} \right\}$$

Find a basis and dim for M.

(2) $$N = \left\{ A : A\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}.$$

Find a basis and dim for N.

A1 $$\begin{pmatrix} a+c+2d & b+c+d \\ -b-c-d & -a-c-2d \end{pmatrix}$$

$$= a\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}_{A} + b\underbrace{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}_{B} + c\underbrace{\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}}_{C} + d\underbrace{\begin{pmatrix} 2 & 1 \\ -1 & -2 \end{pmatrix}}_{D}$$

$$= \text{Span}\left\{ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}, \begin{pmatrix} 2 & 1 \\ -1 & -2 \end{pmatrix} \right\}$$

test for lin dep.

$$a\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + b\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + c\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} + d\begin{pmatrix} 2 & 1 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{aligned} a + c + 2d &= 0 \\ b + c + d &= 0 \\ -b - c - d &= 0 \\ -a - c - 2d &= 0 \end{aligned} \quad \Rightarrow \quad \left(\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & -1 & -1 & -1 & 0 \\ -1 & 0 & -1 & -2 & 0 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Basis $= \{A, B\}$

Dim $= 2$

A2 Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$A\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 3a+4b \\ 3c+4d \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left.\begin{aligned} 3a + 4b &= 0 \\ 3c + 4d &= 0 \end{aligned}\right\} \Longrightarrow \begin{aligned} b &= \text{free}, \quad a = -\tfrac{4}{3}b \\ d &= \text{free}, \quad c = -\tfrac{4}{3}d \end{aligned}$$

$$A = \begin{pmatrix} -\frac{4}{3}b & b \\ -\frac{4}{3}d & d \end{pmatrix} = b\underbrace{\begin{pmatrix} -\frac{4}{3} & 1 \\ 0 & 0 \end{pmatrix}}_{B_1} + d\underbrace{\begin{pmatrix} 0 & 0 \\ -\frac{4}{3} & 1 \end{pmatrix}}_{B_2}$$

(why B_1, B_2 lin ind?)

Basis vectors, dim = 2

Quiz 7 Solution

(a) Find a matrix A such that the linear transform $X \longrightarrow AX$ is onto but not one-to-one.

(b) Find a matrix A such that the linear transform $X \longrightarrow AX$ is one-to-one not onto.

(c) Find a matrix A such that the linear transform $X \longrightarrow AX$ is neither onto nor one-to-one

A $A^{m\times n}$

onto $\iff$ Rank $= m$

not onto $\iff$ Rank $< m$

one-to-one $\iff$ Nullity $= 0$

not one-to-one $\iff$ Nullity > 0

(a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}^{2\times 3}$ Rank $= 2$, Nullity $= 1 > 0$

(b) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}^{3\times 2}$ Rank $= 2 < 3$, Nullity $= 0$

(c) $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}^{2\times 2}$ Rank $= 1 < 2$ Nullity $= 1 > 0$