MA 351 Fall 2024 (Aaron N. K. Yip)
Homework 1
Due: Thursday, Aug. 29, in class

Penney, Linear Algebra: Ideas and Applications (4th edition)
Section 1.2 EXERCISES
p. 38: 1.49, 1.50, 1.51, 1.53, 1.55(a,c,e,g,i,k), 1.59.

Additional Problems: Consider the following transformation T on vectors from $\mathbb{R}^2$:

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

and a, b, c, d are some given numbers. Prove the following linearity statements about T:

(a): $$T\left(\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}\right) = T\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + T\begin{pmatrix} x_2 \\ y_2 \end{pmatrix};$$

(b): $$T\left(\lambda\begin{pmatrix} x \\ y \end{pmatrix}\right) = \lambda T\begin{pmatrix} x \\ y \end{pmatrix};$$

(c): $$T\left(\lambda\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \mu\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}\right) = \lambda T\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \mu T\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}.$$

In the above, λ and μ are some numbers. In fact, statement (c) includes both statement (a) ($\lambda = \mu = 1$) and (b) ($\mu = 0$). For this problem, you need to prove all the statements.