MA 351 Fall 2021 (Aaron N. K. Yip)
Homework 6
Due: Thursday, Oct. 19, in class

Penney, Linear Algebra: Ideas and Applications (4th edition)
p. 108:
2.1, 2.3, 2.6, 2.7 (These are from the Practice Problems before the Test.)
p. 123:
2.32, 2.33 (No need to show that the sets are subspaces. Just find only one set of basis and the dimension for each set. Hint: factor or separate out the free variables, $a, b, c, \ldots$.)
2.35, 2.36,
2.41(a,c) (You are encouraged to also think about parts (b,d,e) but no need to hand in.)
2.43, 2.44,
2.46 (*In addition*, write any vector $aA + bB$ in the form of $xX + yY$.)

1. (Continuation of Additional Problem Homework 5 #1)

   Let $V = \mathbf{R}^+$, the set of *positive real numbers*. On V we define the following "scalar multiplication" and "vector addition":

   (i) (scalar multiplication, $\cdot$) for any $\alpha \in \mathbf{R}$ and $u \in V$, $\alpha \cdot u = u^\alpha$, i.e. raising u to its usual α power; (note: u^α still belongs to V so that the scalar multiplication is a legitimate operation. Hence (i) above is satisfied.)

   (ii) (vector addition, $\oplus$) for any $u, v \in V$, $u \oplus v = uv$, i.e. taking the usual multiplication between u and v. (note: uv still belongs to V so that the vector addition is a legitimate operation. Hence (ii) above is satisfied.)

   In Homework 5, you have already shown that with the above definitions of vector addition and scalar multiplication, V is a vector space.

   To continue, find *a* basis and *the* dimension of V.

   (Hint: given any two vectors u and v from V, can you find a relationship between them?)