MA 351 Fall 2025 (Aaron N. K. Yip)
Homework 8
Due: Friday, Nov. 21, 12:00pm, in MATH 432 (slide under the door if closed)

Penney, Linear Algebra: Ideas and Applications (4th edition)
p.249 EXERCISES: 4.1(acegi, use co-factor expansion), 4.4, 4.5;
p.258 EXERCISES: 4.12(acg, use row-operations), 4.13(acg, use column operations), 4.15, 4.16, 4.17, 4.19

Additional Problem. This problem shows that for *square* matrices A and B, *if* $AB = I$, *then* $BA = I$. In the following, *all matrices are* $n \times n$.

Now, let's start from $AB = I$ and complete the following sequence of thoughts/statements.

1. Show that if $BX = 0$ then X must be the *zero* vector and hence $\text{Nullity}(B) = 0$. (Hint: multiply $BX = 0$ by A from the left hand side.)
2. Why $\text{Rank}(B) = n$?
3. Show that there is a C such that $BC = I$. (Hint: use the fact that $BX = b$ has a solution for *any* b. Choose b to be the standard basis vectors of R^n.)
4. Combining $AB = I$ and $BC = I$, show that $A = C$.
5. Conclusion: $BA = I$.