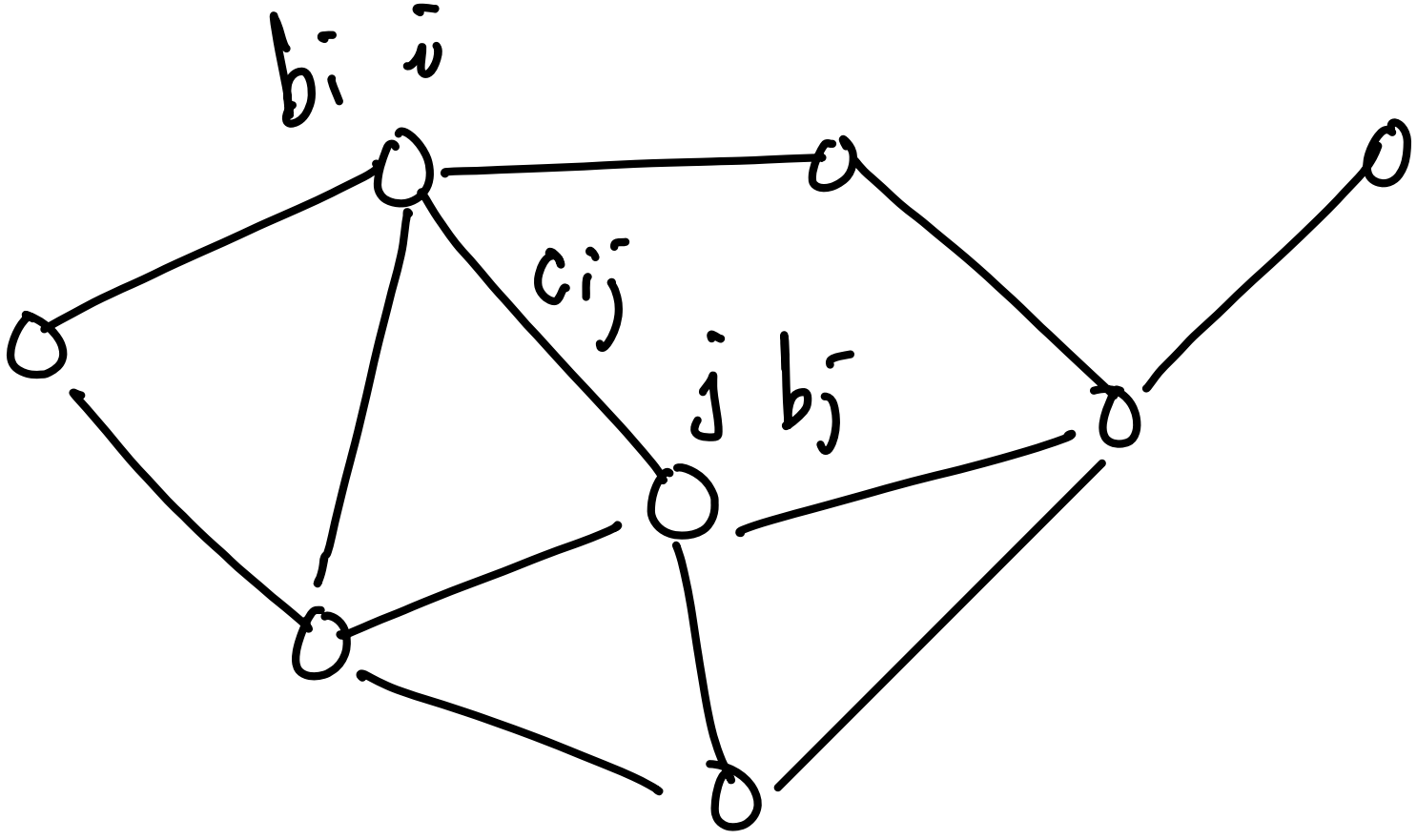
Economic Interpretation of Network Simplex

[C] p. 297.



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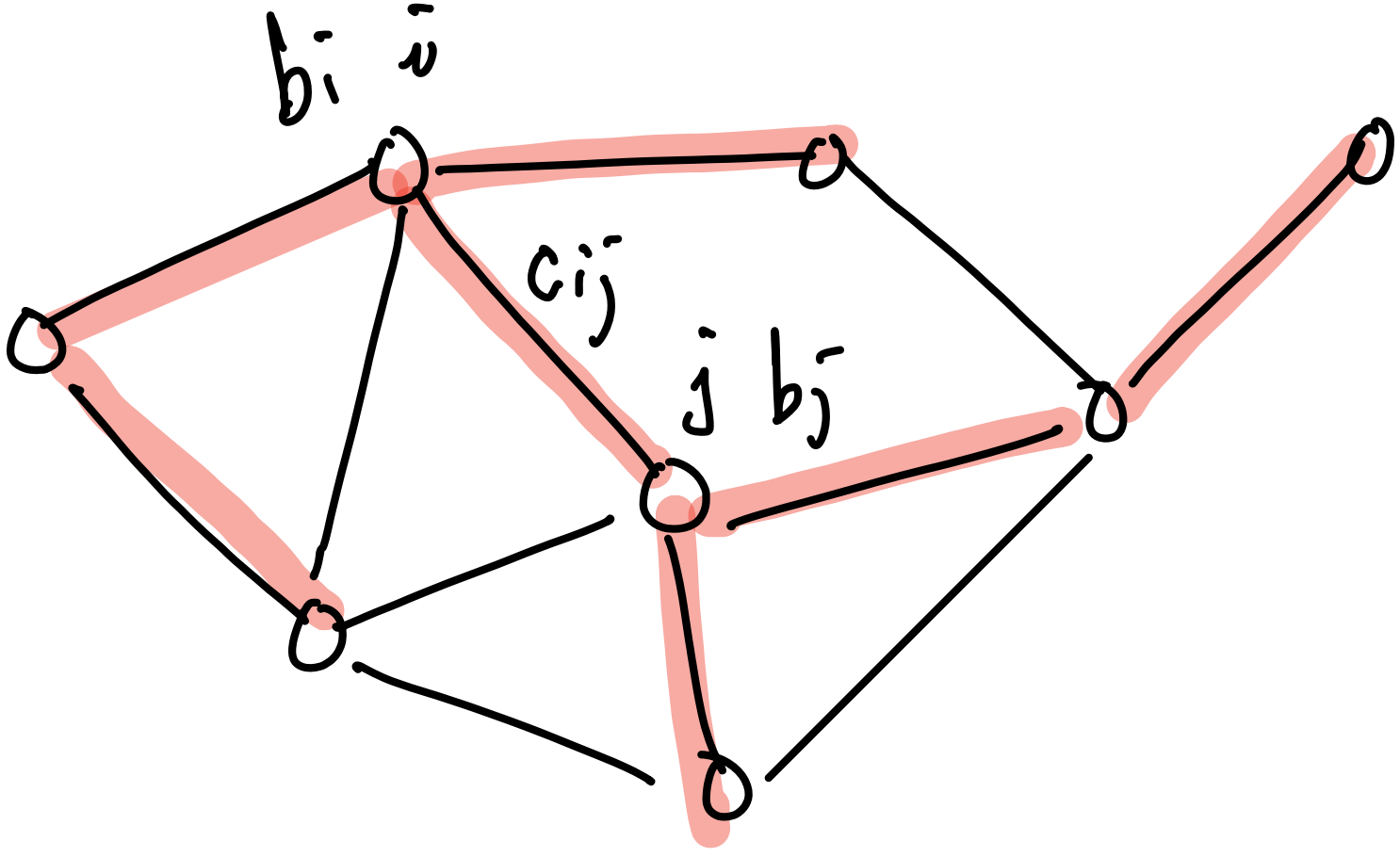
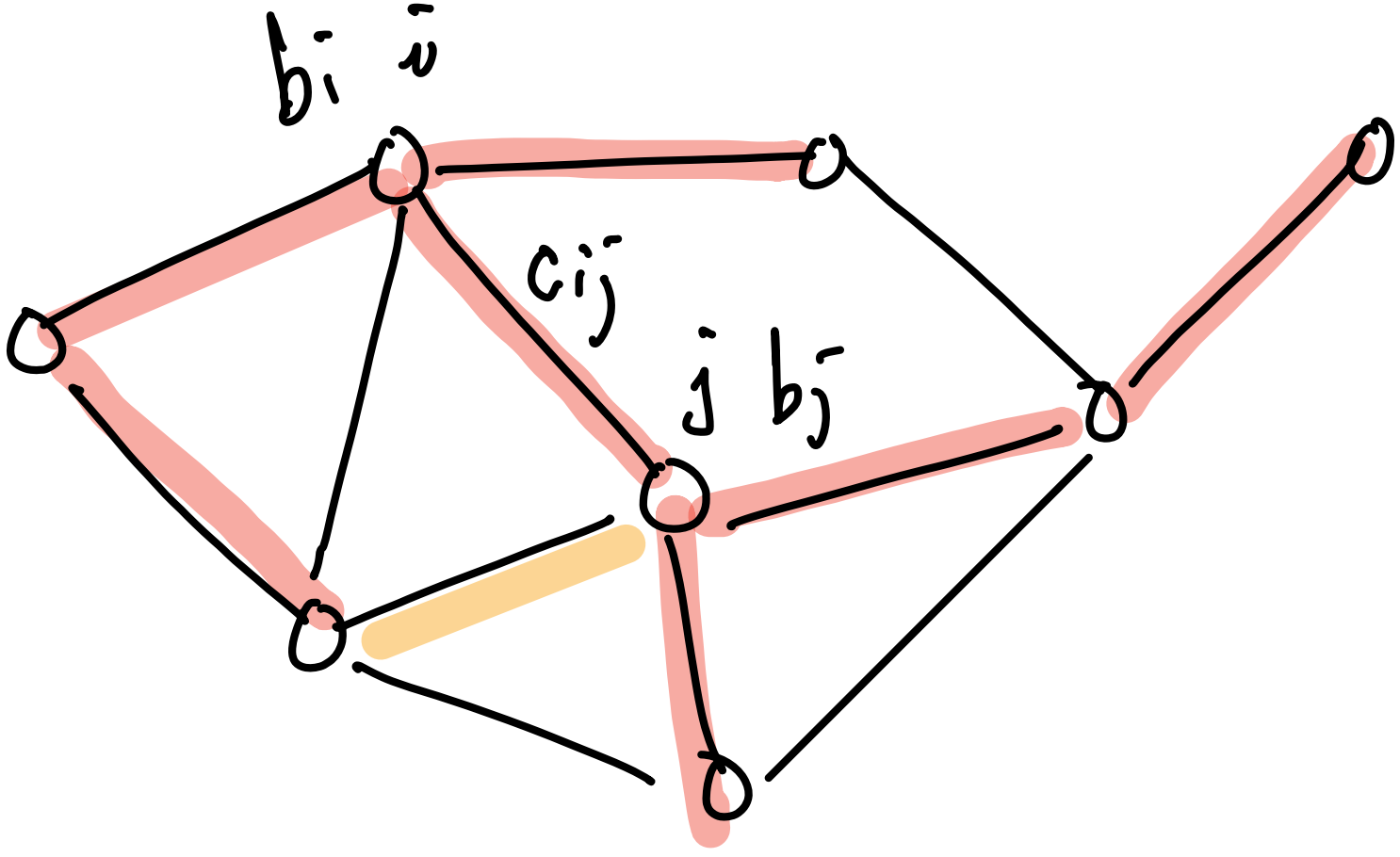


Start a feasible solution on a spanning tree (?)

Economic Interpretation of Network Simplex



(?)

Start a feasible solution on a spanning tree

Economic Interpretation of Network Simplex

Three steps of simplex

(1)

In the first part of the iteration, we imagine that the current feasible tree solution has been adopted by a transportation company. Because of the shipping costs, the market price of the commodity will vary with location: for example, oranges cost more in Alaska than they do in Florida. If the unit price is y_i at a node i and if the company ships the commodity along an arc ij, then it is fair to expect a unit price of $y_i + c_{ij}$ at the node j: a lower price would make shipping along the arc ij wasteful, whereas a higher price would allow competitors to undersell the company at j and still make a profit for themselves. Our first task is to determine a set of "fair prices" $y_1, y_2, \ldots, y_n$ such that

$$y_i + c_{ij} = y_j \qquad \text{for each } ij \in T. \qquad (19.4)$$

tree arc

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Three steps of simplex

(2)

In the second step of each iteration, we put ourselves in the role of a competitor: would it pay to buy the commodity at some node i, ship it along an arc ij and sell at the node j? In our example, the answer is affirmative for several different arcs ij. For instance, we could buy at the node 2 for $y_2 = 23$ and ship along the arc 25 for $c_{25} = 37$: the total expense of $y_2 + c_{25} = 60$ compares favorably with the selling price of $y_5 = 62$. In general, this second step consists of choosing an arc ij such that

$$y_i + c_{ij} < y_j. \tag{19.5}$$

↖ non-tree arc

$z_{ij} < 0$

(i) - - - - - - -> (j)

$y_j - y_i + z_{ij} = c_{ij}$ i.e. $y_j - y_i > c_{ij}$

Economic Interpretation of Network Simplex

Three steps of simplex

(3)

In the third step of each iteration, we imagine that the first company has found out about its competitor's plan and hastens to use the information to its own advantage. It will now ship t units through the entering arc and keep ignoring other out-of-tree arcs; the shipments along the arcs of T will have to be adjusted so as to maintain feasibility of the resulting schedule. The adjustments, easy to work out, are shown in

$$\tilde{z}_{ij} = 0, \text{ i.e. } \tilde{y}_j - \tilde{y}_i = c_{ij}$$