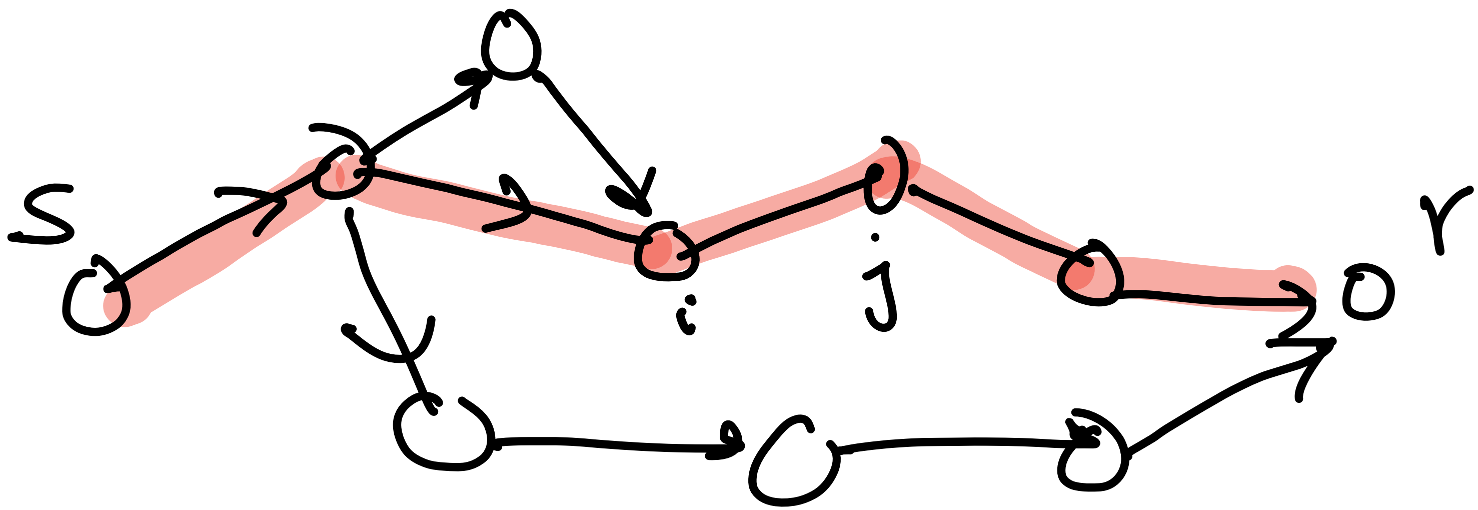
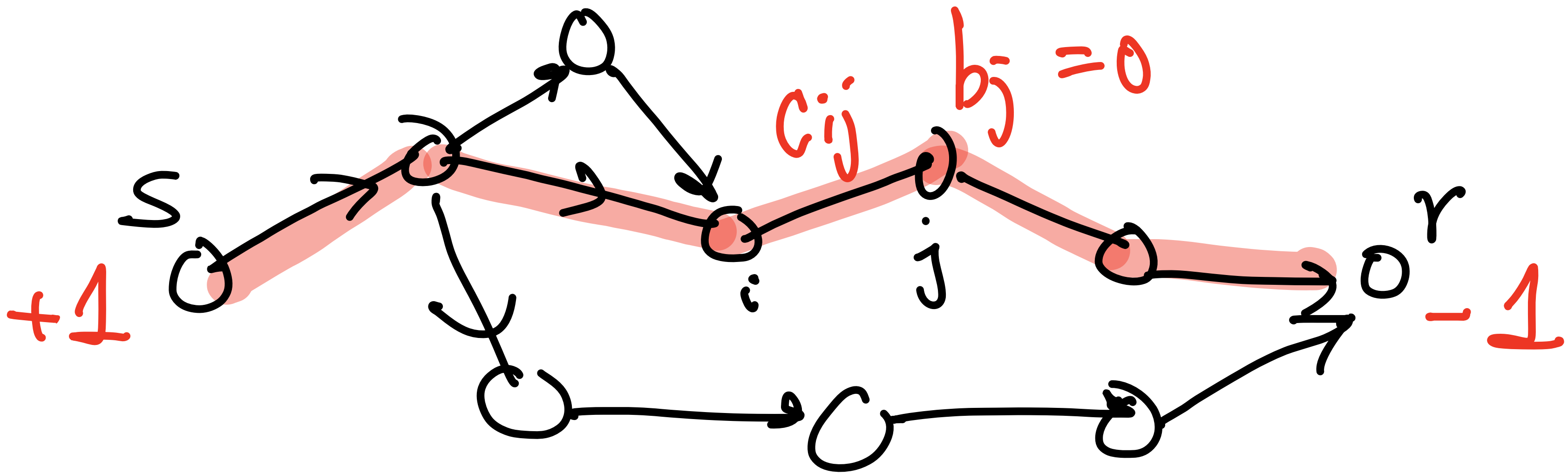
# Shortest Path Problem



# Shortest Path Problem



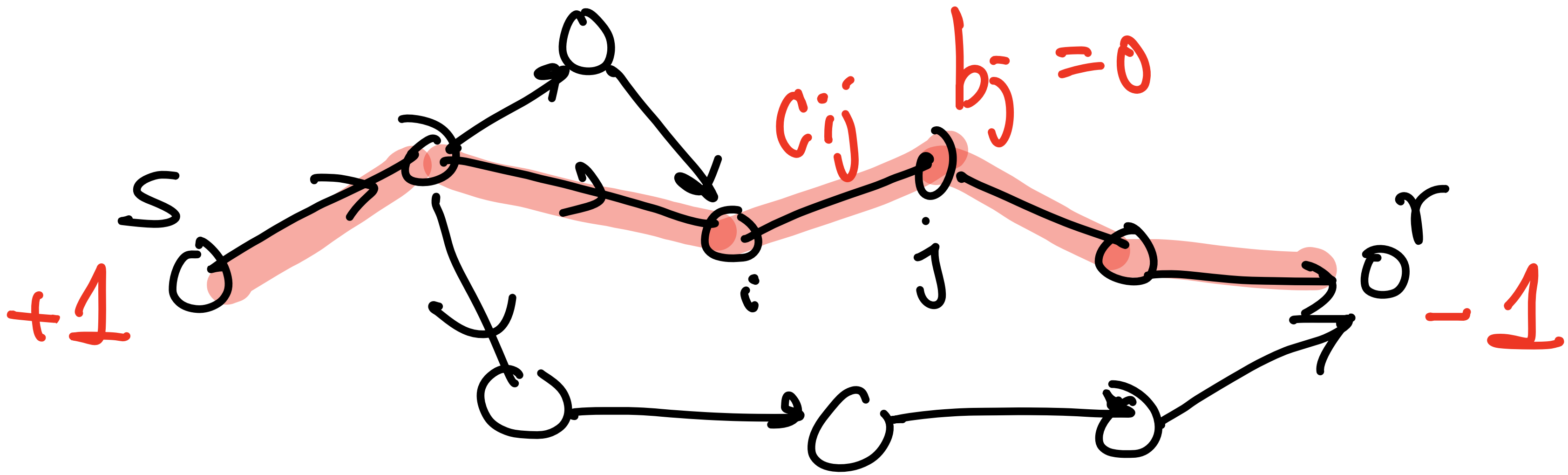

## Network flow formulation

$$\min \sum_{i,j} c_{ij} x_{ij}$$

s.t. $\displaystyle\sum_j x_{sj} = 1, \quad \sum_i x_{ir} = 1$

$(x_{ij} \geq 0)$ $\quad \displaystyle\sum_i x_{ik} - \sum_j x_{kj} = 0, \quad k \neq s, r$

# Shortest Path Problem




## Network flow formulation

$(x_{ij} \in \{0, 1\} ?)$

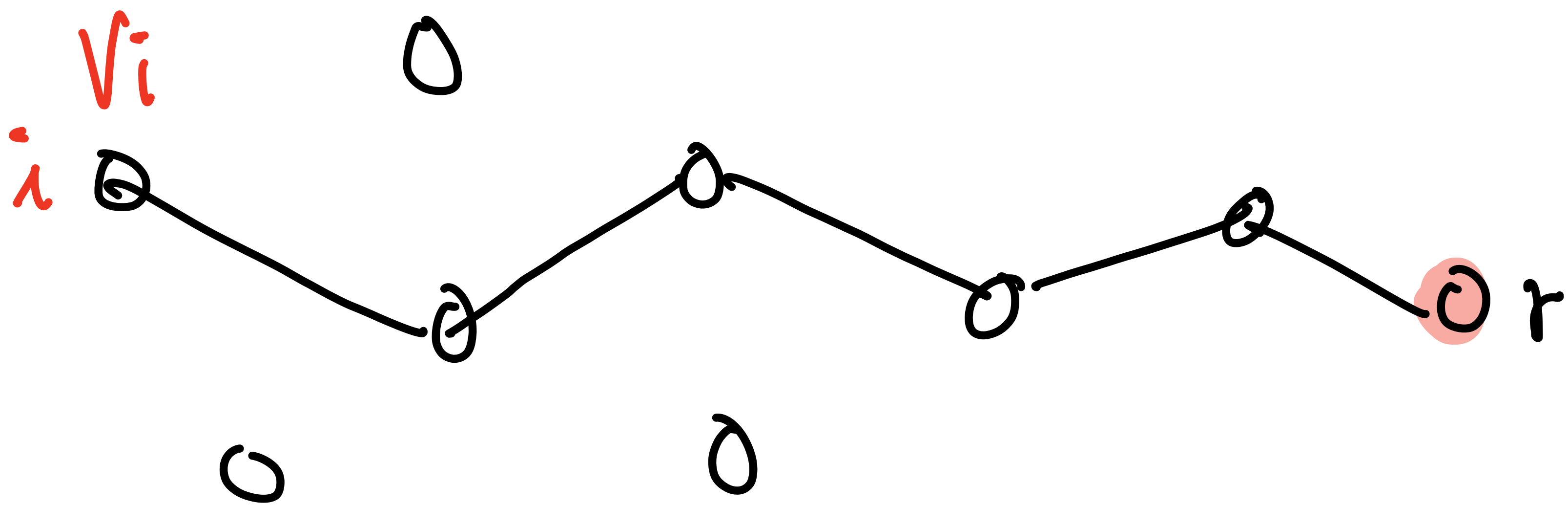
$$\min \sum_{ij} c_{ij} x_{ij}$$

s.t. $\displaystyle\sum_j x_{sj} = 1, \quad \sum_i x_{ir} = 1$

$(x_{ij} \geq 0)$

$$\sum_i x_{ik} - \sum_j x_{kj} = 0, \quad k \neq s, r$$

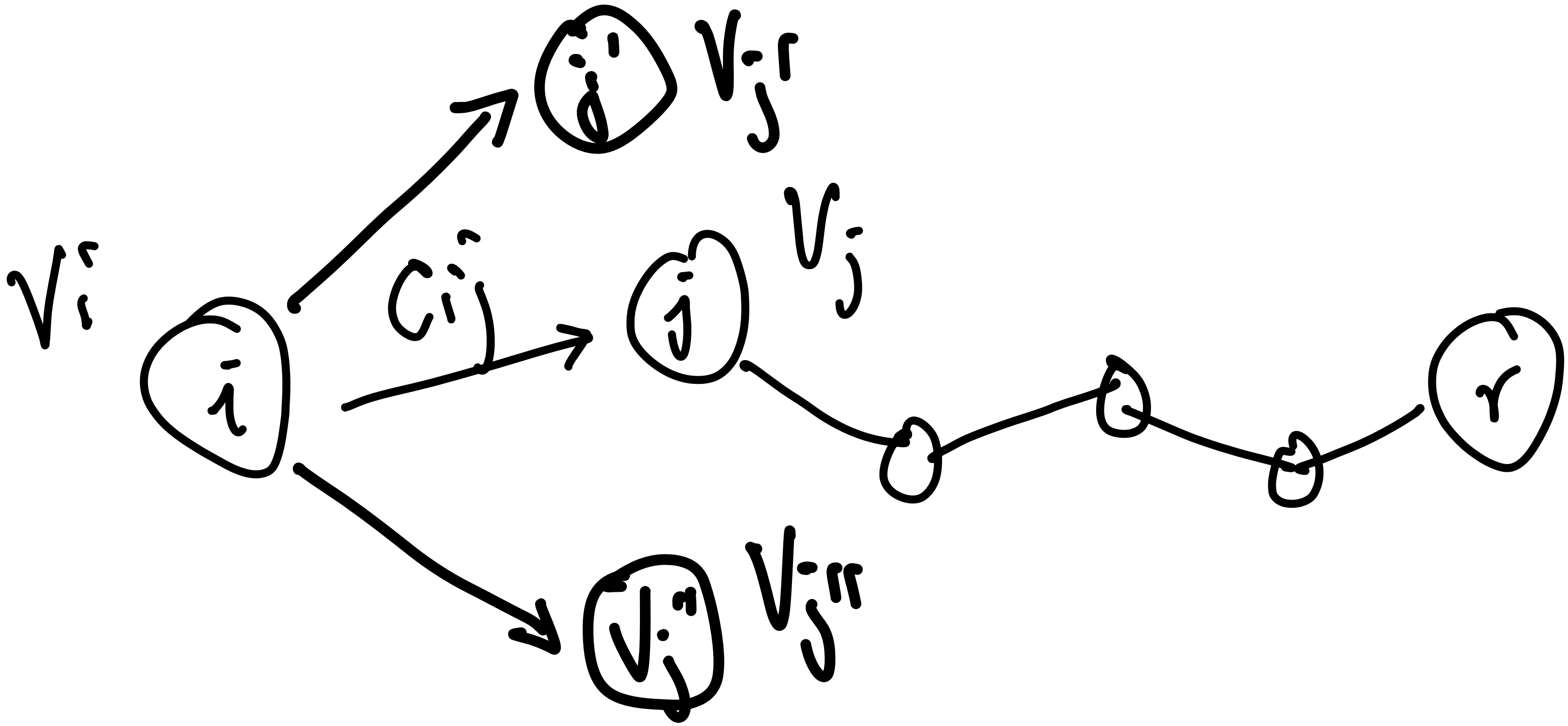
# Shortest Path Problem (Bellman's Eqn)




$$\boxed{V_i = \text{shortest distance from } i \text{ to } r}$$

label

# Shortest Path Problem (Bellman's Eqn)




$$\boxed{\begin{aligned} V_i &= \min_j \{ C_{ij} + V_j \} \\ V_r &= 0 \end{aligned}}$$

# Shortest Path Problem (Bellman's Eqn)

$$\boxed{V_i = \min_j \{C_{ij} + V_j\}, \quad V_r = 0}$$

## Approximation / Iteration

$k=0 \qquad V_r^{(0)} = 0, \qquad V_i^{(0)} = +\infty, \quad i \neq r$

$k \geq 0 \qquad V_r^{(k)} = 0, \qquad V_i^{(k+1)} = \min_j \{C_{ij} + V_j^{(k)}\},$

# Shortest Path Problem (Bellman's Eqn)

$$\boxed{V_i = \min_j \{c_{ij} + V_j\}, \quad V_r = 0}$$

## Approximation / Iteration

$$k = 0 \qquad V_r^{(0)} = 0, \qquad V_i^{(0)} = +\infty, \quad i \neq r$$

$$k \geq 0 \qquad V_r^{(k)} = 0, \quad V_i^{(k+1)} = \min_j \{c_{ij} + V_j^{(k)}\},$$

- $V_i^{(k)}$ = shortest path from $i$ to $r$, with $\leq k$ arcs.
- Needs at most $m$ iterations, $m$ = # of nodes

# Shortest Path Problem (Dijkstra Alg.)

$\mathcal{J}$ – set of finished node

$V_j$ – label of $j$ (shortest distance $j$ to $r$)

$h_j$ – node to visit next in shortest path

# Shortest Path Problem (Dijkstra Alg.)

[V. p.263]

Initialize:

$$\mathcal{F} = \emptyset$$

$$v_j = \begin{cases} 0 & j = r, \\ \infty & j \neq r. \end{cases}$$

while $(|\mathcal{F}^c| > 0)\{$

$\quad j = \operatorname{argmin}\{v_k : k \notin \mathcal{F}\}$

$\quad \mathcal{F} \leftarrow \mathcal{F} \cup \{j\}$

$\quad$ for each $i$ for which $(i, j) \in \mathcal{A}$ and $i \notin \mathcal{F}$ $\{$

$\qquad$ if $(c_{ij} + v_j < v_i)$ $\{$

$\qquad\quad v_i = c_{ij} + v_j$

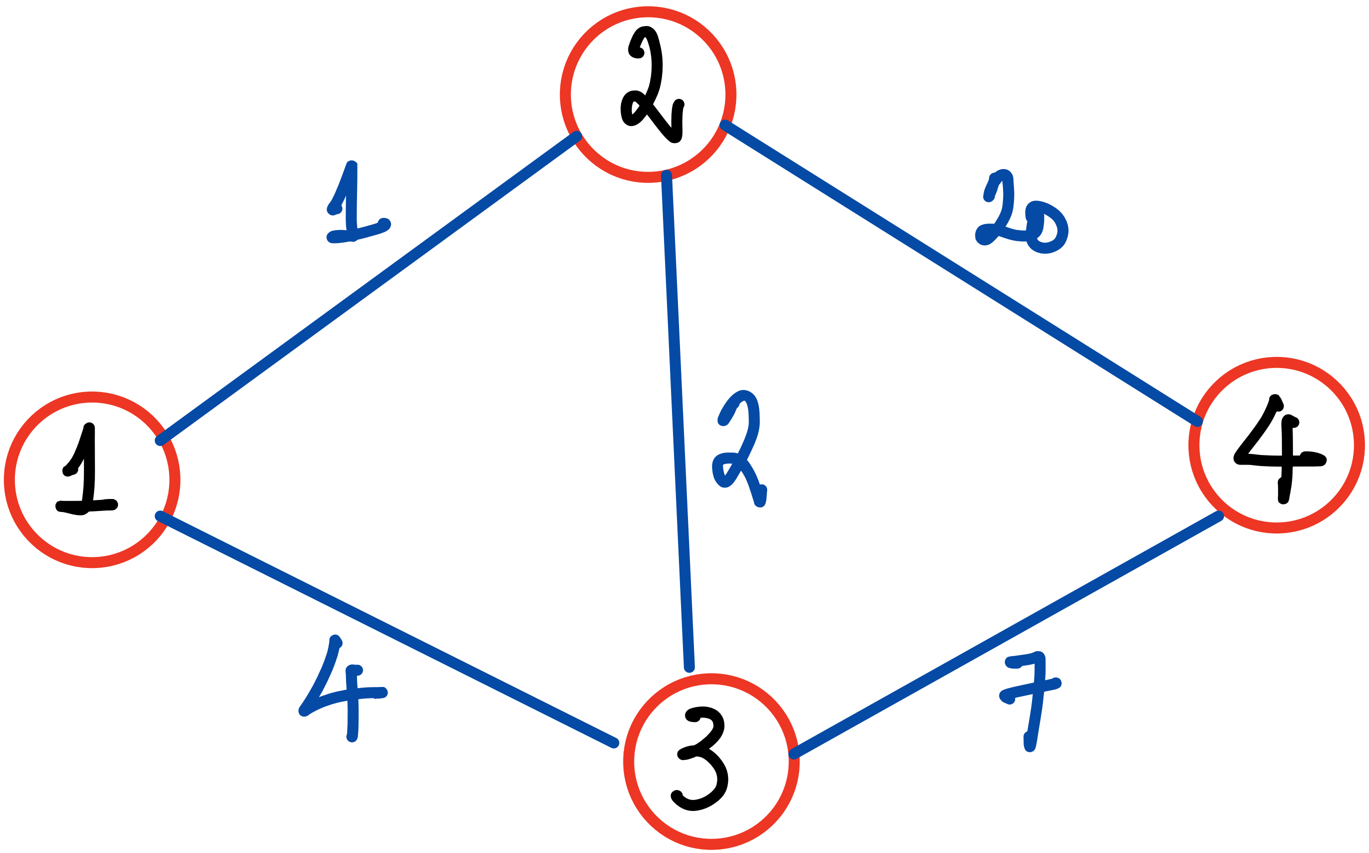
$\qquad\quad h_i = j$

$\qquad \}$

$\quad \}$

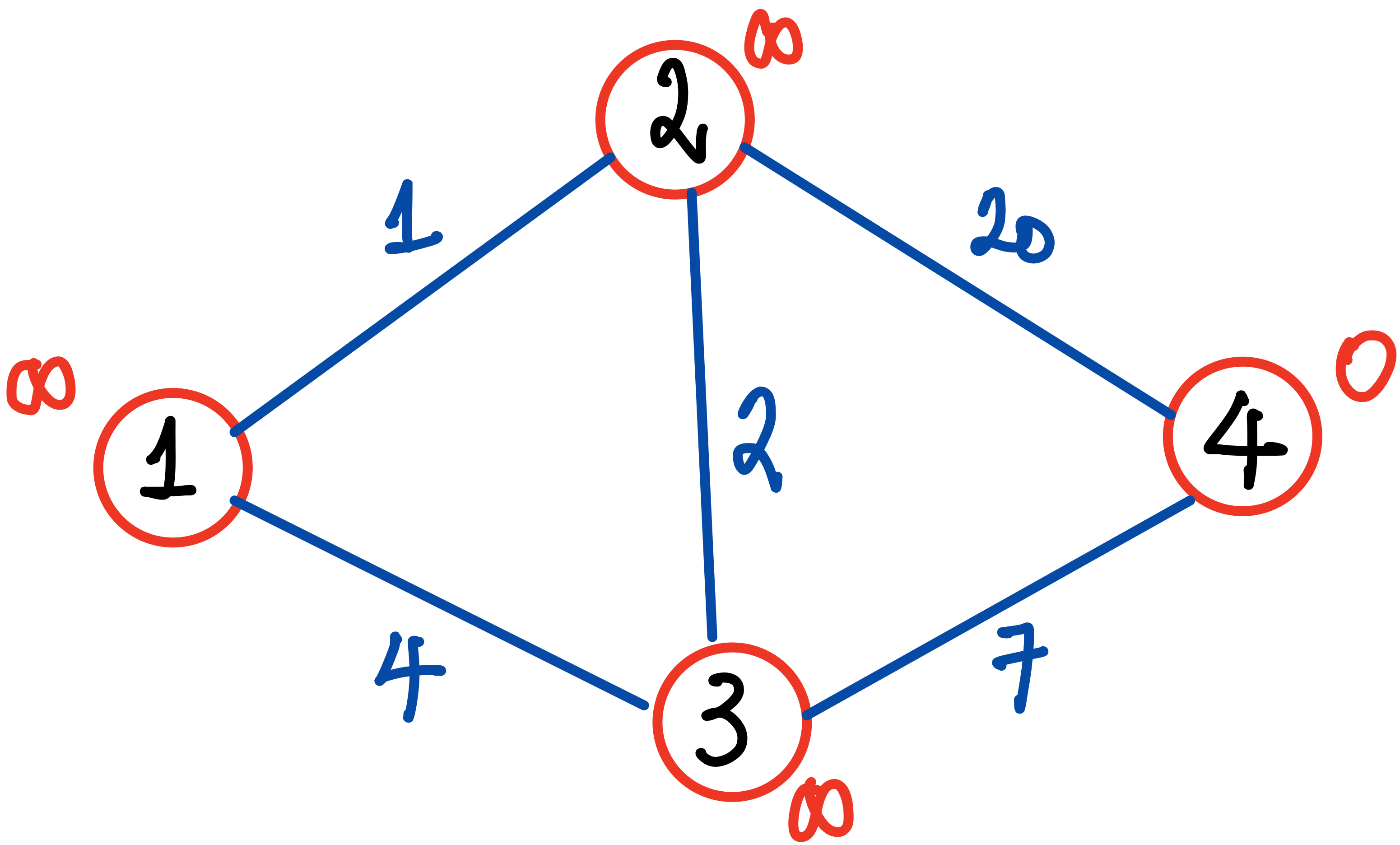
$\}$

# Shortest Path Problem (Dijkstra Alg.)



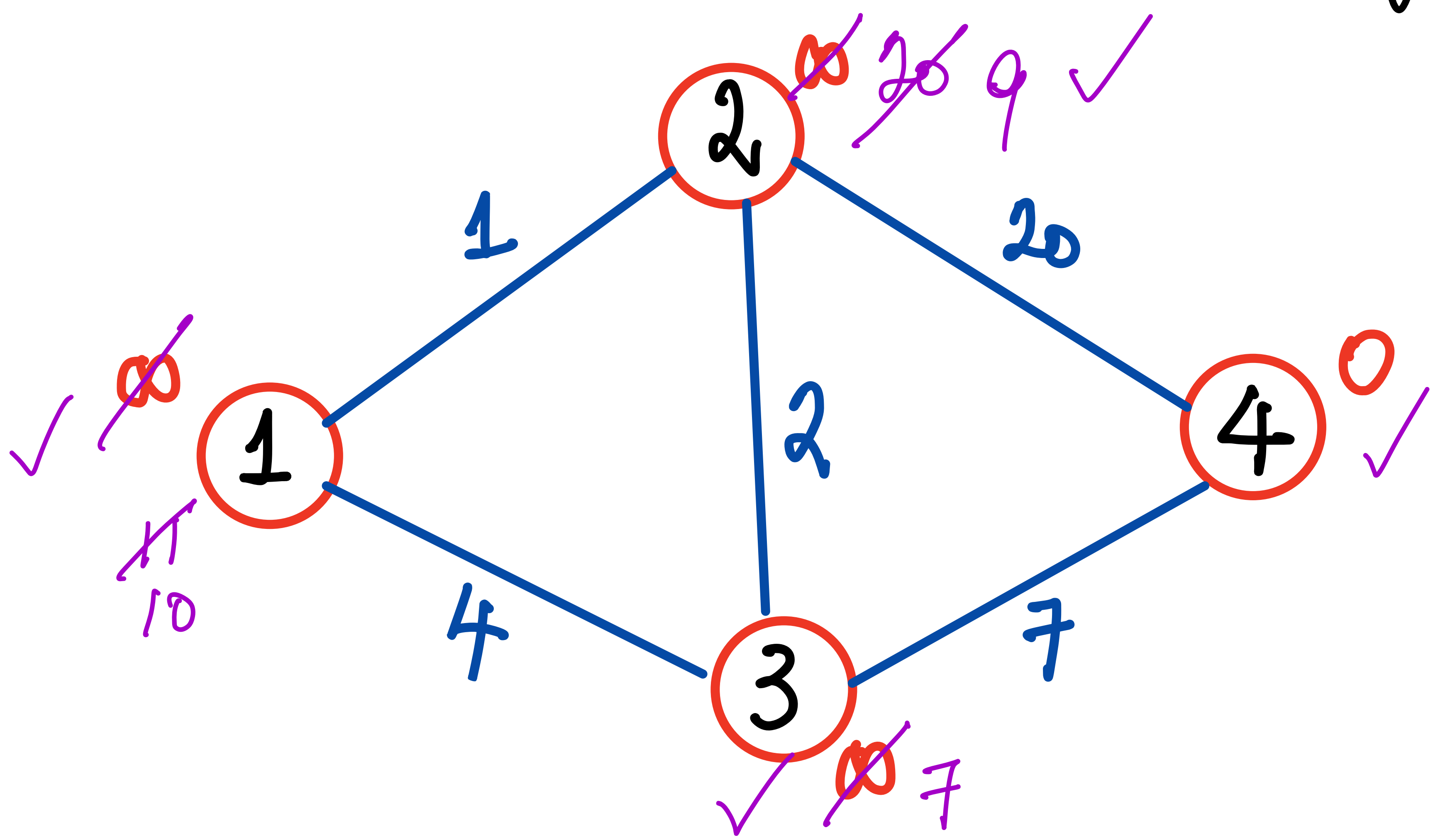

Q: shortest distance between 1 & 4 ?

# Shortest Path Problem (Dijkstra Alg.)




Q: shortest distance between 1 & 4 ?

# Shortest Path Problem (Dijkstra Alg.)




Q: shortest distance between ① & ④ ?