

Simplex in Matrix Form [V, ch. 6]

$$\begin{aligned} \text{(P)} \quad & \max \quad f(x) = c^T x \\ \text{s.t.} \quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

$$\begin{aligned} \text{(D)} \quad & \max \quad -\xi(y) = -b^T y \\ \text{s.t.} \quad & A^T y \geq c \\ & y \geq 0 \end{aligned}$$

$$\begin{aligned} \max \quad & f(x) = f^* - (z_N^*)^T x_N \\ \text{s.t.} \quad & x_B = x_B^* - (\bar{B}^{-1} N) x_N \end{aligned}$$

$$\begin{aligned} \max \quad & -\xi(z) = -f^* - (x_B^*)^T z_B \\ \text{s.t.} \quad & z_N = z_N^* + (\bar{B}^{-1} N)^T z_B \end{aligned}$$

$$\begin{aligned} & Ax \leq b \\ \Leftrightarrow & Ax + w = b \\ \Leftrightarrow & [A \quad I] \begin{bmatrix} x \\ w \end{bmatrix} = b \end{aligned}$$

$$\begin{aligned} & [B \quad N] \begin{bmatrix} x_B \\ x_N \end{bmatrix} = b \\ \Leftrightarrow & Bx_B + Nx_N = b \\ \Leftrightarrow & x_B = \bar{B}^{-1}b - \bar{B}^{-1}Nx_N \end{aligned}$$

Simplex in Matrix Form [V, ch. 6]

$$\begin{aligned} \text{(P)} \quad \max \quad & f(x) = c^T x \\ \text{s.t.} \quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

$$\begin{aligned} \text{(D)} \quad \max \quad & -\xi(y) = -b^T y \\ \text{s.t.} \quad & A^T y \geq c \\ & y \geq 0 \end{aligned}$$

$$\begin{aligned} \max \quad & f(x) = f^* - (z_N^*)^T x_N \\ & x_B = x_B^* - (\bar{B}^{-1} N) x_N \end{aligned}$$

$$\begin{aligned} \max \quad & -\xi(z) = -f^* - (x_B^*)^T z_B \\ \text{s.t.} \quad & z_N = z_N^* + (\bar{B}^{-1} N)^T z_B \end{aligned}$$

$$\begin{aligned} f(x) &= c^T x = c_B^T x_B + c_N^T x_N \\ &= c_B^T (\bar{B}^{-1} b - \bar{B}^{-1} N x_N) + c_N^T x_N \\ &= c_B^T \bar{B}^{-1} b - (c_B^T \bar{B}^{-1} N - c_N^T) x_N \end{aligned}$$

Simplex in Matrix Form [v, ch. 6]

(B - Basic var indices)
(N - Non-Basic var. indices)

$$\max f(x) = f^* - \overbrace{(\bar{B}^{-1}N)^T C_B - C_N}^{Z_N^*} x_N$$
$$x_B = \underbrace{\bar{B}^{-1}b}_{x_B^*} - (\bar{B}^{-1}N)x_N$$

Revised Simplex Method [V, ch.8], [C, ch.7]

Suppose B, N, B^{-1} are known

(1) From z_N^* (>0) $\Rightarrow$ determine entering var.
(e.g. j^{th} col. of $(B^{-1}N)$)

z^*	$-(z_N^*)^T$
x_B^*	$-(B^{-1}N)$

j^{th} col. of $B^{-1}N$
 $= (B^{-1}N)e_j$
 $= B^{-1}(Ne_j)$
 $= \Delta x_B a_j$

Revised Simplex Method [V, ch.8], IC, ch.7]

(2) Find max t s.t. $x_B^* - t \Delta x_B \geq 0$

z^*	$-(z_N^*)^T$
x_B^*	$-(B^{-1}N)$

j^{th} col of $B^{-1}N$
 $= (\bar{B}^{-1}N)e_j$
 $= B^{-1}(Ne_j)$
 $= \Delta x_B a_j$

determine leaving variable.
(eg. i^{th} variable of x_B)

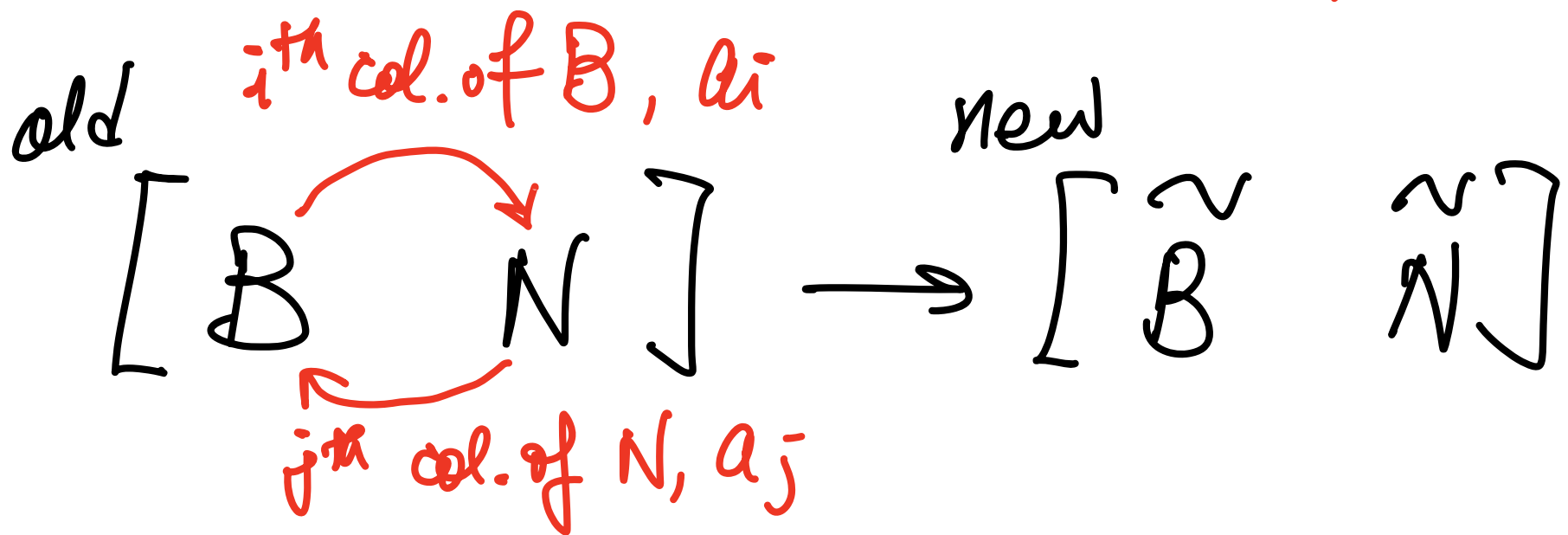
Revised Simplex Method [V, ch.8], IC, ch.7]

(3) Update B, N and B, N

(interchange j^{th} col. of $N = Ne_j = a_j$

and i^{th} var of $X_B \Rightarrow i^{\text{th}}$ col. of B

$$Be_i = a_i, e_i = B^{-1}a_i$$



Revised Simplex Method [V, ch.8], IC, ch.7]

$$\text{old} \begin{bmatrix} B & N \end{bmatrix} \xrightarrow{\text{new}} \begin{bmatrix} \tilde{B} & \tilde{N} \end{bmatrix}$$

$a_i \quad a_j$



$$\tilde{B} = B + (\vec{a}_j - \vec{a}_i) \vec{e}_i^T$$

$$= B(I + B^{-1}(\vec{a}_j - \vec{a}_i) \vec{e}_i^T)$$

$$= B(I + (B^{-1}\vec{a}_j - B^{-1}\vec{a}_i) \vec{e}_i^T)$$

$$= B(I + (B^{-1}N\vec{e}_j - \vec{e}_i) \vec{e}_i^T)$$

$$\begin{pmatrix} N\vec{e}_j = \vec{a}_j \\ B\vec{e}_i = \vec{a}_i \end{pmatrix}$$

Revised Simplex Method [V, ch.8], IC, ch.7]

Suppose B , B^{-1} is known

$$\tilde{B} = B \underbrace{\left(I + (\tilde{B}^{-1} N e_j - e_i) e_i^T \right)}_E$$

$$\tilde{B}^{-1} = \underbrace{\left(I + (\tilde{B}^{-1} N e_j - e_i) e_i^T \right)^{-1}}_E \underbrace{B^{-1}}_{\text{already known}}$$

E^{-1} : easy to compute

Revised Simplex Method [V, ch.8], [C, ch.7]

PROPOSITION 8.1. Given two column vectors u and v for which $1 + v^T u \neq 0$,

$$(I + uv^T)^{-1} = I - \frac{uv^T}{1 + v^T u}.$$

(Proof by direct verification)

PROOF. The proof is trivial. We simply multiply the matrix by its supposed inverse and check that we get the identity:

$$\begin{aligned}(I + uv^T) \left(I - \frac{uv^T}{1 + v^T u} \right) &= I + uv^T - \frac{uv^T}{1 + v^T u} - \frac{uv^T uv^T}{1 + v^T u} \\ &= I + uv^T \left(1 - \frac{1}{1 + v^T u} - \frac{v^T u}{1 + v^T u} \right) \\ &= I,\end{aligned}$$

Revised Simplex Method [V, ch.8], [C, ch.7]

Proof by computation

Recall: $(1-x)^{-1} = 1+x+x^2+x^3+\dots \quad (|x| < 1)$

or $(1+x)^{-1} = 1-x+x^2-x^3+\dots$

$$(I + uv^T)^{-1} = I - uv^T + (uv^T)^2 - (uv^T)^3 + \dots$$

Let $X = uv^T$, $X^2 = uv^T uv^T = \underbrace{(v^T u)}_{\substack{\text{a number}}} \underbrace{uv^T}_X = cX$

Hence $X^3 = c^2 X$, $X^4 = c^3 X \dots$

$$\begin{aligned}(I + uv^T)^{-1} &= I - X + cX - c^2 X + c^3 X - \dots \\ &= I - (1 - c + c^2 - c^3 \dots) X \\ &= I - \frac{1}{1+c} X = I - \frac{uv^T}{1+v^T u}\end{aligned}$$

Revised Simplex Method [V, ch.8], [C, ch.7]

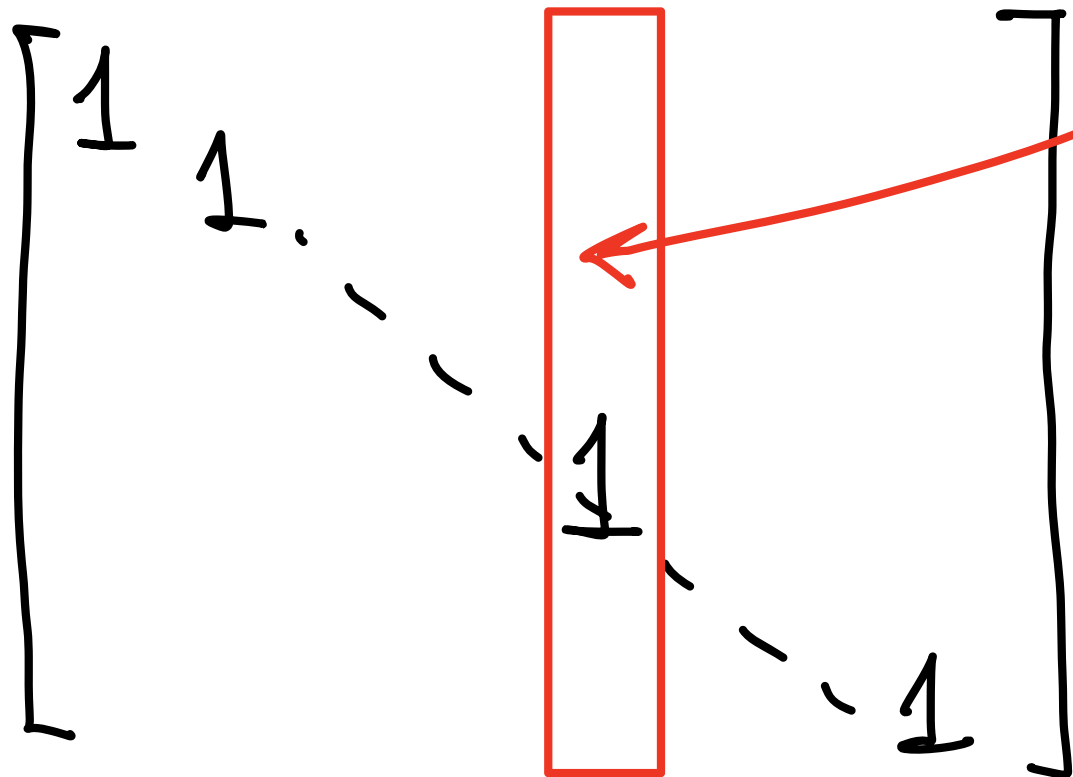
$$E^{m \times m} = \begin{bmatrix} 1 & & \boxed{\alpha_1} & & \\ & \ddots & \alpha_2 & & \\ & & \vdots & & \\ & & \alpha_i & & \\ & & \vdots & & \\ & & \alpha_m & & \ddots & 1 \end{bmatrix}, \quad E^{-1} = \begin{bmatrix} 1 & & \boxed{-\alpha_1/\alpha_i} & & \\ & \ddots & \vdots & & \\ & & \vdots & & \\ & & \vdots & & \\ & & \boxed{-\alpha_m/\alpha_i} & & \ddots & 1 \end{bmatrix}$$

eg

$$\begin{bmatrix} 1 & 0 & \boxed{3} & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 5 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & \boxed{-3/2} & 0 \\ 0 & 1 & -4/2 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & \boxed{-5/2} & 1 \end{bmatrix}$$

Revised Simplex Method [V, ch.8], IC, ch.7]

$$E = I + (\bar{B}^{-1}Ne_j - e_i)e_i^T = I - \underbrace{e_i e_i^T}_{\text{replace } i^{\text{th}} \text{ col. of } I \text{ by } \bar{B}^{-1}Ne_j = j^{\text{th}} \text{ col. of } \bar{B}^{-1}N} + \bar{B}^{-1}Ne_j e_i^T$$



[v] p.95

Example

①

$$\text{maximize } 4x_1 + 3x_2$$

$$\text{subject to } x_1 - x_2 \leq 1$$

$$2x_1 - x_2 \leq 3$$

$$x_2 \leq 5$$

$$x_1, x_2 \geq 0.$$

$$\begin{aligned} x_1 - x_2 + x_3 &= 1 \\ 2x_1 - x_2 + x_4 &= 3 \\ x_2 + x_5 &= 5 \end{aligned}$$

The matrix A is given by

$$\begin{matrix} & \textcolor{red}{1} & \textcolor{red}{2} & \textcolor{red}{3} & \textcolor{red}{4} & \textcolor{red}{5} \\ \begin{bmatrix} 1 & -1 & 1 & & \\ 2 & -1 & & 1 & \\ 0 & 1 & & & 1 \end{bmatrix} \end{matrix}.$$

(Note that some zeros have not been shown.) The initial sets of basic and nonbasic indices are

$$\mathcal{B} = \{3, 4, 5\}$$

and

$$\mathcal{N} = \{1, 2\}.$$

Corresponding to these sets, we have the submatrices of A :

$$B = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

$$N = \begin{bmatrix} 1 & -1 \\ 2 & -1 \\ 0 & 1 \end{bmatrix}.$$

①

$$\begin{aligned}x_1 &\leq 1 \\x_1 &\leq 3/2 \\x_1 &\leq \infty\end{aligned}$$



$$\begin{aligned}\text{maximize } & 4x_1 + 3x_2 \\ \text{subject to } & x_1 - x_2 \leq 1 \\ & 2x_1 - x_2 \leq 3 \\ & x_2 \leq 5 \\ & x_1, x_2 \geq 0.\end{aligned}$$

x_1 enters, largest coeff.

$$\begin{aligned}x_1 - x_2 + x_3 &= 1 \\ 2x_1 - x_2 + x_4 &= 3 \\ x_2 + x_5 &= 5\end{aligned}$$

The matrix A is given by

$$\begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{bmatrix} 1 & -1 & 1 & 0 & 0 \\ 2 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

x_3 leaves

(Note that some zeros have not been shown.) The initial sets of basic and nonbasic indices are

$$\underline{\mathcal{B} = \{3, 4, 5\}} \quad \text{and} \quad \underline{\mathcal{N} = \{1, 2\}}.$$

Corresponding to these sets, we have the submatrices of A :

$$B = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \quad N = \begin{bmatrix} 1 & -1 \\ 2 & -1 \\ 0 & 1 \end{bmatrix}.$$

Hint

Undo

Number format: Fraction

Zero Visibility: Visible

Current Dictionary

maximize $\zeta =$ 4 x_1 + 3 x_2

subject to:

$$\begin{array}{rclclcl}
 w_1 & = & 1 & - & 1 & x_1 & - & -1 & x_2 \\
 w_2 & = & 3 & - & 2 & x_1 & - & -1 & x_2 \\
 w_3 & = & 5 & - & 0 & x_1 & - & 1 & x_2
 \end{array}$$

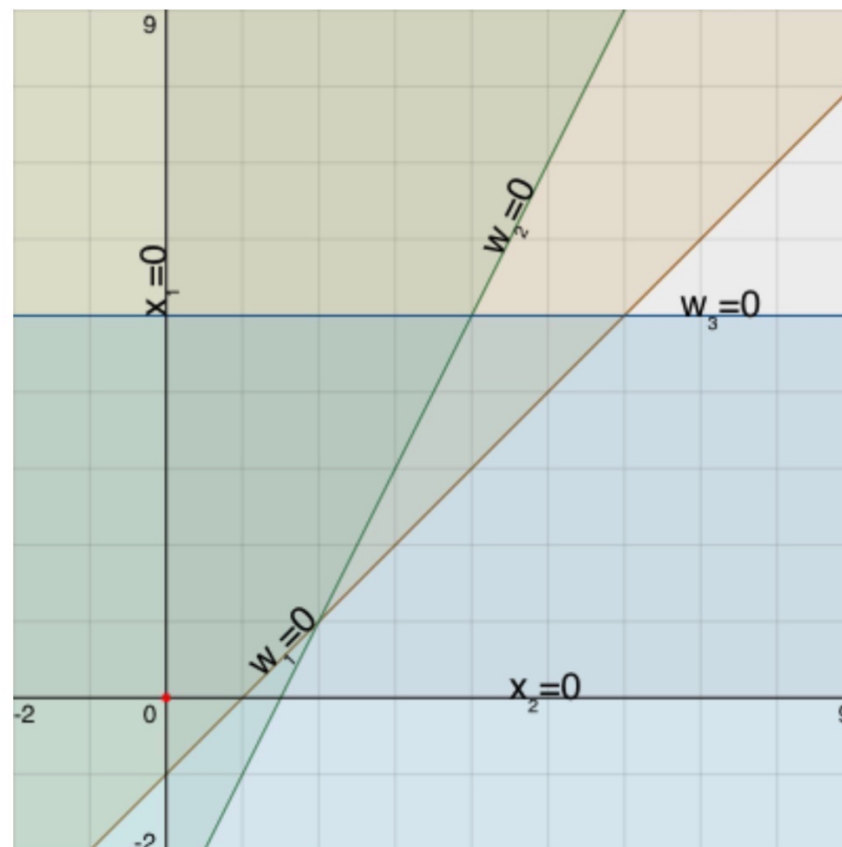
 $x_1, x_2, w_1, w_2, w_3 \geq 0$

Optimal

Infeasible

Unbounded

Pick a judge: Bart Simpson



$$B_0 = I,$$

$$B_0^{-1}b = I^{-1}b = b$$

1st col of $B^{-1}N$

Compare

Step 9. The new sets of basic and nonbasic indices are

②

$$\mathcal{B} = \{1, 4, 5\} \quad \text{and} \quad \mathcal{N} = \{3, 2\}.$$

Corresponding to these sets, we have the new basic and nonbasic submatrices of A ,

$$B = \begin{bmatrix} 1 & & \\ 2 & 1 & \\ 0 & & 1 \end{bmatrix} \quad N = \begin{bmatrix} 1 & -1 \\ 0 & -1 \\ 0 & 1 \end{bmatrix},$$

and the new basic primal variables and nonbasic dual variables:

$$x_{\mathcal{B}}^* = \begin{bmatrix} x_1^* \\ x_4^* \\ x_5^* \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix} \quad z_{\mathcal{N}}^* = \begin{bmatrix} z_3^* \\ z_2^* \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \end{bmatrix}.$$

x_2 enters
 x_4 leaves

$$\begin{bmatrix} 1 & -1 & 1 & & \\ 2 & -1 & & 1 & \\ 0 & 1 & & & 1 \end{bmatrix}$$

1 2 3 4 5

maximize $\zeta = 4 + (-4)w_1 + 7x_2$

subject to:

$x_1 =$	1	-	1	w_1	-	-1	x_2
$w_2 =$	1	-	-2	w_1	-	1	x_2
$w_3 =$	5	-	0	w_1	-	1	x_2

$$x_1 \quad x_2 \quad w_1 \quad w_2 \quad w_3 \geq 0$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} 4 \\ -7 \end{pmatrix} = \underline{(B^{-1}N)^T} C_B - C_N = \left(\underline{\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} 1 & -1 \\ 0 & -1 \\ 0 & 1 \end{bmatrix} \right)^T \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix} = \underline{B^{-1}} b = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$$

$$\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \underline{\text{2nd col. of } B^{-1}N} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

enters

Step 9. The new sets of basic and nonbasic indices are

②

$$\mathcal{B} = \{1, 4, 5\} \quad \text{and} \quad \mathcal{N} = \{3, 2\}.$$

Corresponding to these sets, we have the new basic and nonbasic submatrices of A ,

$$B = \begin{bmatrix} 1 & & \\ 2 & 1 & \\ 0 & & 1 \end{bmatrix} \quad N = \begin{bmatrix} 1 & -1 \\ 0 & -1 \\ 0 & 1 \end{bmatrix},$$

and the new basic primal variables and nonbasic dual variables:

$$x_{\mathcal{B}}^* = \begin{bmatrix} x_1^* \\ x_4^* \\ x_5^* \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix} \quad z_{\mathcal{N}}^* = \begin{bmatrix} z_3^* \\ z_2^* \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \end{bmatrix}.$$

x_2 enters
 x_4 leaves

$$\begin{bmatrix} 1 & -1 & 1 & & \\ 2 & -1 & & 1 & \\ 0 & 1 & & & 1 \end{bmatrix}$$

1 2 3 4 5

maximize $\zeta = 4 + (-4)w_1 + 7x_2$

subject to:

x_1	=	1	-	1	w_1	-	-1	x_2
w_2	=	1	-	-2	w_1	-	1	x_2
w_3	=	5	-	0	w_1	-	1	x_2

$$x_1 \quad x_2 \quad w_1 \quad w_2 \quad w_3 \geq 0$$

$$B = \{1, 2, 5\}, N = \{3, 4\}$$

← 2nd col. of $B^T N$

$$B^{\text{new}} = \begin{bmatrix} 1 & -1 & 0 \\ 2 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = B^{\text{old}} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$(B^{\text{new}})^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ -2 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}$$

3

Step 9. The new sets of basic and nonbasic indices are

$$\mathcal{B} = \{1, 2, 5\}$$

and

$$\mathcal{N} = \{3, 4\}.$$

Corresponding to these sets, we have the new basic and nonbasic submatrices of A ,

$$B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$$

and the new basic primal variables and nonbasic dual variables:

$$x_{\mathcal{B}}^* = \begin{bmatrix} x_1^* \\ x_2^* \\ x_5^* \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} \quad z_{\mathcal{N}}^* = \begin{bmatrix} z_3^* \\ z_4^* \end{bmatrix} = \begin{bmatrix} -10 \\ 7 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & -1 & 1 & & \\ 2 & -1 & & 1 & \\ 0 & 1 & & & 1 \end{bmatrix}$$

Current Dictionary

$$\begin{array}{ll} \text{maximize} & \zeta = 11 + 10 w_1 + -7 w_2 \\ \text{subject to:} & x_1 = 2 - -1 w_1 - 1 w_2 \\ & x_2 = 1 - -2 w_1 - 1 w_2 \\ & w_3 = 4 - 2 w_1 - -1 w_2 \end{array}$$

$$x_1 \quad x_2 \quad w_1 \quad w_2 \quad w_3 \geq 0$$

$$\begin{pmatrix} -10 \\ 7 \end{pmatrix} = (\underline{B^T N})^T C_B - C_N = \left(\begin{bmatrix} -1 & 1 & 0 \\ -2 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 2 \end{bmatrix} \right)^T \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = \underline{B^T b} = \begin{bmatrix} -1 & 1 & 0 \\ -2 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

$$\begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix} = \underline{1^{st} \text{ col. of } B^T N} = \begin{bmatrix} -1 & 1 & 0 \\ -2 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

3

Step 9. The new sets of basic and nonbasic indices are

$\mathcal{B} = \{1, 2, 5\}$ and $\mathcal{N} = \{3, 4\}$.

Corresponding to these sets, we have the new basic and nonbasic submatrices of A ,

$$B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$$

and the new basic primal variables and nonbasic dual variables:

$$x_{\mathcal{B}}^* = \begin{bmatrix} x_1^* \\ x_2^* \\ x_5^* \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} \quad z_{\mathcal{N}}^* = \begin{bmatrix} z_3^* \\ z_4^* \end{bmatrix} = \begin{bmatrix} -10 \\ 7 \end{bmatrix}.$$

x_3 enters
 x_5 leaves

$$\begin{bmatrix} 1 & -1 & 1 & & \\ 2 & -1 & & 1 & \\ 0 & 1 & & & 1 \end{bmatrix}$$

Current Dictionary

maximize	ζ	=	11	+	10	w_3	+	-7	w_4
subject to:	x_1	=	2	-	-1	w_1	-	1	w_2
	x_2	=	1	-	-2	w_1	-	1	w_2
	w_3	=	4	-	2	w_1	-	-1	w_2

$$x_1 \quad x_2 \quad w_1 \quad w_2 \quad w_3 \geq 0$$

Opt.

Step 9. The new sets of basic and nonbasic indices are

$$\mathcal{B} = \{1, 2, 3\}$$

and

$$\mathcal{N} = \{5, 4\}.$$

Corresponding to these sets, we have the new basic and nonbasic submatrices of A ,

$$B = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad N = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix},$$

Optimal
as

$$z_N^* \geq 0$$

$$(x_B^* \geq 0)$$

and the new basic primal variables and nonbasic dual variables:

$$x_B^* = \begin{bmatrix} x_1^* \\ x_2^* \\ x_3^* \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 2 \end{bmatrix}$$

$$z_N^* = \begin{bmatrix} z_5^* \\ z_4^* \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}.$$

$$\delta^* = 31$$

Current Dictionary

$$\begin{array}{ll} \text{maximize} & \zeta = 31 + (-5)w_1 + (-2)w_2 \\ \text{subject to:} & x_1 = 4 - \frac{1}{2}w_3 - \frac{1}{2}w_2 \\ & x_2 = 5 - 1w_3 - 0w_2 \\ & w_1 = 2 - \frac{1}{2}w_3 - \frac{1}{2}w_2 \end{array}$$

$$x_1 \quad x_2 \quad w_1 \quad w_2 \quad w_3 \geq 0$$

1st col. of $B^{-1}N$
↓

$$B^{\text{new}} = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix} = B^{\text{old}} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ 2 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{aligned} B^{-1} &= \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & -1 & 0 \\ 2 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}^{-1} \\ &= \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ -2 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \\ 1 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \end{aligned}$$

$$\begin{pmatrix} 5 \\ 2 \end{pmatrix} = (\bar{B}^T N)^T C_B - C_N$$

$$= \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \\ 1 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}^T \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \quad \checkmark$$

$$\begin{pmatrix} 4 \\ 5 \\ 2 \end{pmatrix} = \bar{B}^{-1} \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \\ 1 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 2 \end{bmatrix} \quad \checkmark$$

$$31 = C_B^T \bar{B}^{-1} b = (4 \ 3 \ 0) \begin{pmatrix} 4 \\ 5 \\ 2 \end{pmatrix} = 31 \quad \checkmark$$

Hint

Undo

Number format: Fraction Zero Visibility: Visible

Current Dictionary

maximize $\zeta = 31 + (-5)w_1 + (-2)w_2$

subject to:

$x_1 = 4 - \frac{1}{2}w_3 - \frac{1}{2}w_2$
$x_2 = 5 - 1w_3 - 0w_2$
$w_1 = 2 - \frac{1}{2}w_3 - \frac{1}{2}w_2$

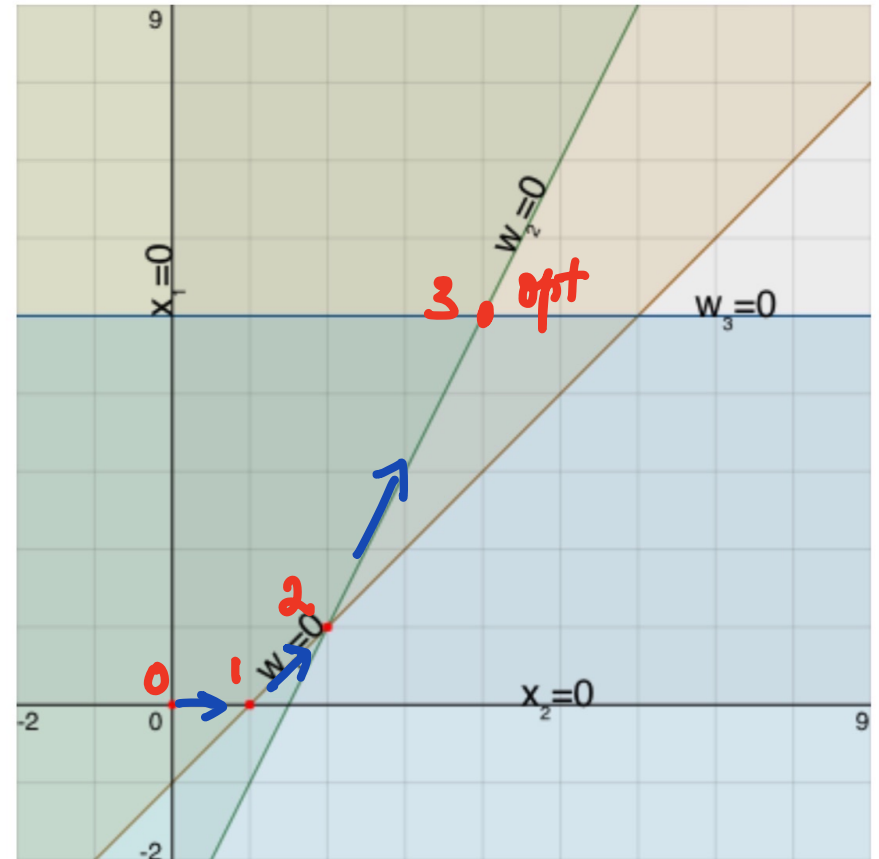
$$x_1 \quad x_2 \quad w_1 \quad w_2 \quad w_3 \geq 0$$

Optimal

Infeasible

Unbounded

Pick a judge: Bart Simpson



$$\zeta^* = 31 \quad (= 4x_1^* + 3x_2^* = 4(4) + 3(5))$$