MA 421 Fall 2026 Review for Midterm

Linear Programming

- Simplex
  - Dictionary
  - Basic vs non-basic vars.
  - Phase I, II
- Dual
  - Weak Duality Thm
  - Strong Duality Thm
  - Complementary Slackness
  - Dual base Phase I
- Simplex in Matrix-form

$$\max \zeta = c^T x \quad \text{s.t.} \quad Ax \le b,\ x \ge 0$$

$$\Longleftrightarrow$$

$$\max \zeta = c^T x + 0^T w \quad Ax + w = b,\ x, w \ge 0$$

$$Ax + w = b \iff \begin{bmatrix} A & I \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} = b$$

$$B, N \implies \begin{bmatrix} B & N \end{bmatrix} \begin{bmatrix} x_B \\ x_N \end{bmatrix} = b$$

$$B x_B + N x_N = b$$

$w = b - Ax$ $\qquad$ $x_B = B^{-1} b - B^{-1} N x_N$

≥ 0 (feasibility)

$$z = \begin{bmatrix} c^T & 0^T \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} = \begin{bmatrix} c_B^T & c_N^T \end{bmatrix} \begin{bmatrix} x_B \\ x_N \end{bmatrix}$$

$$= c_B^T x_B + c_N^T x_N$$

$$= c_B^T \left[B^{-1} b - B^{-1} N x_N \right] + c_N^T x_N$$

$$= c_B^T B^{-1} b - \left(c_B^T B^{-1} N - c_N^T \right) x_N$$

$$= b^T B^{-T} c_B - \left((B^{-1} N)^T c_B - c_N \right)^T x_N$$

≥ 0 for optimality

- Sensitivity Analysis

Nonlinear Programming

- Unconstrained Opt.
- Constrained Op

  Equality: $g(x) = 0$

  Lagrange multiplier

  Inequality $h(x) \leq 0$

  - $h(x) < 0$ interior pt.
  - $h(x) = 0$ boundary pt.

- Inner Product → Projection, Orthogonality
- Least square solution

  $AX = b \Rightarrow A^TAX = A^Tb$

- Min Norm solution
- Quadratic Programming