# Tableau & Dictionary Method

in Linear Programming — a worked tutorial

## Part 1 — Conceptual Comparison

***Q: Can you explain the difference between the Tableau method and Dictionary method in solving linear programming problems?***

Both methods solve the same problem — finding the optimal vertex of a feasible polytope by pivoting between basic feasible solutions — but they organise the information differently and use different notational conventions.

**Setup.** Maximise $z = 5x_1 + 4x_2$ subject to:

```
6x1 + 4x2 <= 24
x1 + 2x2 <= 6
x1, x2 >= 0
```

Adding slack variables $s_1$, $s_2$ converts inequalities to equalities.

### The Dictionary Method

The dictionary writes each *basic variable* explicitly as a function of the *non-basic variables*, plus a separate objective row. At the initial basis $\{s_1, s_2\}$:

```
s1 = 24 - 6x1 - 4x2
s2 = 6 - x1 - 2x2
-----------------
z = 5x1 + 4x2
```

The basic variables ($s_1$, $s_2$) are on the left; non-basic variables ($x_1$, $x_2$, set to zero) are on the right. The current BFS is read off immediately: $s_1 = 24$, $s_2 = 6$, $z = 0$.

### The Tableau Method

The tableau stores exactly the same information as a *matrix*, without writing variable names repeatedly. Rows = constraint equations + objective; columns = variables.

| Basis | x1 | x2 | s1 | s2 | RHS |
|---|---|---|---|---|---|
| **s1** | 6 | 4 | 1 | 0 | 24 |
| **s2** | 1 | 2 | 0 | 1 | 6 |
| **-z** | -5 | -4 | 0 | 0 | 0 |

Most negative entry in -z row: -5 under x1. Ratio test selects row 1.

## Side-by-side comparison

| Feature | Dictionary | Tableau |
|---|---|---|
| Representation | Explicit equations for each basic variable | Matrix; variable names implicit from column positions |
| Pivot operation | Algebraic substitution | Elementary row operations |
| Optimality check | No positive coefficients in z row | No negative entries in -z row |
| Reading solution | Constant terms on right-hand side | RHS column of basic-variable rows |
| Dual variables | Requires extra bookkeeping | Shadow prices from optimal objective row |
| Sensitivity | Awkward; must re-derive | Reads directly from optimal tableau |
| Best for | Learning / understanding simplex | Implementation / large problems |

**Key insight.** The two methods are mathematically identical — same pivot decisions, same sequence of basic feasible solutions. The dictionary is the tableau written in longhand. The tableau's -z row is the negative of the dictionary's objective expression; the RHS column of the tableau equals the dictionary's constant terms.

## Part 2 — Worked 3×4 Example (4 decision variables, 3 constraints)

***Q: Can you solve a same 3x4 example in both dictionary and tableau method?***

**Problem.** Maximise:

```
z = 3x1 + 2x2 + 5x3 + 4x4
```

Subject to:

```
2x1 + x2 + 3x3 + x4 <= 20 ... (C1)
x1 + 3x2 + x3 + 2x4 <= 30 ... (C2)
x1 + x2 + 2x3 + 3x4 <= 24 ... (C3)
x1, x2, x3, x4 >= 0
```

Adding slack variables $s_1$, $s_2$, $s_3$ gives 7 variables and 3 equality constraints.

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# Dictionary Method

## Initial Dictionary — Basis: {s1, s2, s3}

```
s1 = 20 - 2x1 - x2 - 3x3 - x4
s2 = 30 - x1 - 3x2 - x3 - 2x4
s3 = 24 - x1 - x2 - 2x3 - 3x4
------------------------------
z = 3x1 + 2x2 + 5x3 + 4x4
```

BFS: s1=20, s2=30, s3=24, z=0.

### *Pivot 1 selection*

Entering variable: $x_3$ has the largest objective coefficient (5).

Ratio test (RHS / positive x3 coefficient):

```
s1 row: 20/3 = 6.67 <-- minimum => s1 leaves
s2 row: 30/1 = 30
s3 row: 24/2 = 12
```

⇒ ***Enter x3, leave s1. Solve s1 row for x3 and substitute.***

## Dictionary 2 — Basis: {x3, s2, s3}

```
x3 = 20/3 - (2/3)x1 - (1/3)x2 - (1/3)x4 - (1/3)s1
s2 = 70/3 - (1/3)x1 - (8/3)x2 - (5/3)x4 + (1/3)s1
s3 = 32/3 + (1/3)x1 - (1/3)x2 - (7/3)x4 + (2/3)s1
------------------------------
z = 100/3 - (1/3)x1 + (1/3)x2 + (7/3)x4 - (5/3)s1
```

BFS: x3=20/3, s2=70/3, s3=32/3, z=100/3 ≈ 33.3.

### *Pivot 2 selection*

Entering variable: $x_4$ has the largest positive coefficient in z (7/3).

Ratio test:

```
x3 row: (20/3)/(1/3) = 20
s2 row: (70/3)/(5/3) = 14
s3 row: (32/3)/(7/3) = 32/7 ≈ 4.57 <-- minimum => s3 leaves
```

⇒ ***Enter x4, leave s3. Solve s3 row for x4 and substitute.***

## Dictionary 3 — Basis: {x3, s2, x4} — OPTIMAL

```
x3 = 36/7 - (5/7)x1 - (2/7)x2 - (3/7)s1 + (1/7)s3
s2 = 110/7 - (4/7)x1 -(17/7)x2 - (1/7)s1 + (5/7)s3
x4 = 32/7 + (1/7)x1 - (1/7)x2 + (2/7)s1 - (3/7)s3
---
z = 44 - s1 - s3
```

No positive coefficients in the z row. **Optimal solution found.** x1=0, x2=0, x3=36/7, x4=32/7, z=**44**.

## Tableau Method

### Initial Tableau — Basis: {s1, s2, s3}

| Basis | x1 | x2 | x3 | x4 | s1 | s2 | s3 | RHS |
|---|---|---|---|---|---|---|---|---|
| **s1** | 2 | 1 | 3* | 1 | 1 | 0 | 0 | 20 |
| **s2** | 1 | 3 | 1 | 2 | 0 | 1 | 0 | 30 |
| **s3** | 1 | 1 | 2 | 3 | 0 | 0 | 1 | 24 |
| **-z** | -3 | -2 | -5* | -4 | 0 | 0 | 0 | 0 |

Most negative in -z row: -5 under x3. Ratio test: 20/3=6.67*, 30/1=30, 24/2=12. Pivot element = 3 (row 1, x3 col). * marks pivot.

### *Row operations after Pivot 1:*

```
R1 <-- R1 / 3
R2 <-- R2 - 1*R1
R3 <-- R3 - 2*R1
Rz <-- Rz + 5*R1
```

### Tableau 2 — Basis: {x3, s2, s3}

| Basis | x1 | x2 | x3 | x4 | s1 | s2 | s3 | RHS |
|---|---|---|---|---|---|---|---|---|
| **x3** | 2/3 | 1/3 | 1 | 1/3 | 1/3 | 0 | 0 | 20/3 |
| **s2** | 1/3 | 8/3 | 0 | 5/3* | -1/3 | 1 | 0 | 70/3 |
| **s3** | -1/3 | 1/3 | 0 | 7/3* | -2/3 | 0 | 1 | 32/3 |
| **-z** | -1/3 | -1/3 | 0 | -7/3* | 5/3 | 0 | 0 | -100/3 |

Most negative in -z row: -7/3 under x4. Ratio test (positive x4 entries only): (20/3)/(1/3)=20, (70/3)/(5/3)=14, (32/3)/(7/3)=32/7 ≈ 4.57*. Pivot element = 7/3 (row 3, x4 col).

### *Row operations after Pivot 2:*

```
R3 <-- (3/7)*R3
R1 <-- R1 - (1/3)*R3
R2 <-- R2 - (5/3)*R3
Rz <-- Rz + (7/3)*R3
```

### Tableau 3 — Basis: {x3, s2, x4} — OPTIMAL

| Basis | x1 | x2 | x3 | x4 | s1 | s2 | s3 | RHS |
|---|---|---|---|---|---|---|---|---|
| **x3** | 5/7 | 2/7 | 1 | 0 | 3/7 | 0 | -1/7 | 36/7 |
| **s2** | 4/7 | 17/7 | 0 | 0 | 1/7 | 1 | -5/7 | 110/7 |
| **x4** | -1/7 | 1/7 | 0 | 1 | -2/7 | 0 | 3/7 | 32/7 |

| Basis | x1 | x2 | x3 | x4 | s1 | s2 | s3 | RHS |
|---|---|---|---|---|---|---|---|---|
| **-z** | 0 | 0 | 0 | 0 | 1 | 0 | 1 | -44 |

All entries in -z row are >= 0. Optimal! Read RHS of basic variable rows.

**Optimal solution (both methods agree):**
x1 = 0, x2 = 0, x3 = 36/7 ≈ 5.14, x4 = 32/7 ≈ 4.57, **z = 44**

Verification: 3(0) + 2(0) + 5(36/7) + 4(32/7) = 180/7 + 128/7 = 308/7 = 44 ✓
Slack s2 = 110/7 > 0 => constraint 2 is not binding at optimum.
s1 = s3 = 0 => constraints 1 and 3 are tight (binding).

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## Summary

Both methods made the **same two pivot decisions**: enter $x_3$ first (largest objective coefficient), then enter $x_4$ (largest remaining positive coefficient), reaching optimality in exactly two iterations.

| Aspect | Dictionary | Tableau |
|---|---|---|
| Pivot mechanics | Substitute expression into all equations | Scale pivot row; eliminate via row ops |
| Basis tracking | Left-hand side of each equation | Row label column + identity submatrix |
| Intermediate values | All variable values visible at every step | Read RHS of basic-variable rows |
| Error risk | Algebra mistakes in substitution | Arithmetic errors in row scaling |
| Best use | Building intuition, learning simplex | Computation, sensitivity analysis, software |

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*Note: Mathematical expressions use standard ASCII notation for maximum PDF compatibility. Fractions such as 36/7 represent exact rational values. Asterisks (*) in tableau cells mark the pivot element.*