

MA 421 Fall 2025 (Aaron N. K. Yip)
Homework 1, due on Thursday, Sept 4th, in class

1. [V] (*Linear Programming, Foundations and Extensions*, **5th edition**)
p.8 Exercises 1.1, 1.2;
p.57 Exercises 4.9 (Read [V, p.45] for a motivation of this problem. One method of solution seems to be by mathematical induction, though I haven't tried it myself.)
2. Posted note in Week 1: (*First Course in Linear Optimization*, Beck, Guttman-Beck)
p.29 Exercises 1.2
3. Suppose you live in a farm and own a couple of cows which can produce 22 gallons of milk each week. As a side job or hobby, you plan to make use of the cows' milk to produce some dairy products (ice-cream and butter) for sale in the local farmer's market. The following are the specifics for your decision.

You need 3 gallons of milk to make 1 gallon of ice cream and 2 gallons of milk to produce 1 kilogram of butter. You own a huge refrigerator that can store practically unlimited amounts of butter, but your freezer can hold at most 6 gallons of ice cream. In order not to affect your full time job, you can work at most 6 hours per week to manufacture these delicious products. One hour of work is needed to produce either 4 gallons of ice cream or 1 kilogram of butter. (Any fraction of one hour is needed to produce the corresponding fraction of product.)

From historical cost-benefit analysis, you set the prices to ensure a profit of \$5 per gallon of ice cream and \$4 per kilogram of butter.

(a) How much ice cream and butter you should produce in order to maximize your profit? (Assume you have established such a good reputation that you are always able to sale everything you bring to the farmer's market.)

(b) Suppose you plan to change the price of ice-cream. What's the range for the new price so that the optimal solution is the same as the previous part?

(c) Now your neighbor – who also owns some cows but does not have the expertise or interest to manufacture the product – would like to participate in your endeavor by selling his cow milk for \$1 per gallon. Should you take your neighbor's offer? If so, how much milk you can afford to buy? (Assume all the previous specifics remain the same as (a).)

(You can use purely graphical method to solve this problem.)