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MA 421 Fall 2025 (Aaron N. K. Yip)

Homework 4, due on Thursday, Sept 25th, 11:59pm, in Gradescope

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1. [V] (*Linear Programming, Foundations and Extensions*, **5th edition**)

   p.81: #5.1, #5.6[1]

2. Redo problems [V] #2.10, #3.4, [C] #1.2 by considering their dual problems.

3. [C] p.69, #5.3.

4. In class[2], we have verified that for the LP problem in [C], p.67, the suggested solution $(x_1^* = 25, x_2^* = 75)$ is indeed optimal.

   (a) *Without solving the original LP problem*, write down the optimal dictionary for this problem.

   (b) From (a), write down the optimal solution of the dual problem.

5. Consider the following system of inequalities:

$$
\begin{aligned}
x_1 + x_2 &\leq 2; \\
x_2 &\geq x_1 + 1; \\
x_2 &\leq 2x_1 - 6; \\
x_1, x_2 &\geq 0.
\end{aligned}
$$

   Prove that the above system has no solution, i.e. there is no point (x_1, x_2) satisfying the above equalities, by means of the following three methods:

   (a) Graphical illustration;

   (b) Phase I simplex using the auxiliary variable x_0 [V, p.17];

   (c) Dual based Phase I [V, p.71].

   (You can convert the above problem into an LP problem by introducing an arbitrary (but convenient) objective function.)

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[1]I keep finding more and more "similarities" between [V] and [C]. Among others, [V] #5.6 is the same as [C] p.69 #5.2. I should make a tally of these...

[2]See p.19-20 of the posted note "Complementary slackness" in Week 4.