**MA 421 Fall 2025 (Aaron N. K. Yip)**
**Homework 8, due on Thursday, Dec 4th, 11:59pm, in Gradescope**
**(Project paper due on Friday, Dec 5th, 11:59pm, in Brightspace)**

1. [V]:
   p.168, Exercises: 10.2;
   p.414, Exercises: 23.4, 23.5

2. [C]:
   p.248, 16.10. (For each part, just do the "easy" direction of the "if and only if".)

3. In Homework 7, you were asked to find a separating hyperplane for two disjoint polyhedrons. Now you will do the same but with one of the polyhedrons being a "point". In particular, consider,

$$\begin{aligned} P_1 &= \{x_1 \geq 0;\ \ x_2 \geq \frac{x_1}{2} + 5;\ \ x_1 + x_2 \geq 10\}; \\ P_2 &= \{x_1 = -1,\ \ x_2 = 7\}. \end{aligned}$$

   By means of Farkas Lemma, find a hyperplane (a straight line in this case) that separates $P_1$ and $P_2$. Use your favorite graphing software to plot $P_1$, $P_2$ and the hyperplane in one single graph.

4. [CZ]:

   **21.2** Find local extremizers for:

   **a.** $x_1^2 + x_2^2 - 2x_1 - 10x_2 + 26$ subject to $\frac{1}{5}x_2 - x_1^2 \leq 0$, $5x_1 + \frac{1}{2}x_2 \leq 5$.

   **b.** $x_1^2 + x_2^2$ subject to $x_1 \geq 0$, $x_2 \geq 0$, $x_1 + x_2 \geq 5$.

   For each of the above parts, plot the feasible region, apply KKT and find the values of all the variables and the Lagrange multipliers corresponding to the extremizers.

5. Given the following points in $\mathbb{R}^2$:

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \end{pmatrix}.$$

   Use the method of quadratic programming to find the smallest circle enclosing all of the above points. Use your favorite graphic package to plot the points and the circle you have found.

   (Hint: read pages 8-9 of the note Quadratic Programming from Week 12.)