The Physics of Kettledrums
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The Physics of Kettledrums

The vibrations of a violin string form a harmonic series with a distinct pitch. The vibrations of an ideal membrane do not form such a series. How, then, can a kettledrum have a pitch?

by Thomas D. Rossing

As musically inclined spectators at a parade with a marching band can testify, the bass drum makes a sound that is loud but has no pitch. This is true not only of the small bass drums, 26 to 28 inches in diameter, that are heard in marching bands but also of the substantially larger bass drums, 36 to 40 inches in diameter, heard in a concert orchestra. It is true also of the snare drum, the tenor drum and many other kinds of drum. In contrast, another group of drums, the kettledrums or timpani, convey a strong sense of pitch. So too do still other kinds of drum, such as the tabla of India. What is it about drums that divides them into such different families?

In answering this question one must begin by describing a theoretical abstraction. What all true drums (a grouping that excludes such percussion instruments as steel drums and hollow-log drums) have in common is a drumhead (or drumheads) consisting of a stretched membrane. How would an ideal membrane vibrate? Musically any membrane, real or ideal, can be considered as being analogous to the string of a stringed instrument except that a string essentially has only the one dimension of length and a membrane has the two dimensions of length and breadth. With both a string and a membrane the "restoring" force that makes the system vibrate after applied pressure has deformed it is tension. With a string the tension is applied linearly and is adjusted by means of a tuning peg; the greater the tension, the higher the string's fundamental note. With a circular membrane such as a drumhead the tension is applied around the circumference. Like a string, a membrane can be tuned by altering its tension.

There is one major difference, however, between the vibrations of an ideal string and those of an ideal membrane. The frequencies of a string's overtones are harmonics: integral multiples of the fundamental frequency. With the ideal membrane these "mode" frequencies are not harmonics. Furthermore, the nodes of a vibrating string, that is, the positions along its length that remain essentially stationary while the rest of the string is in motion, are in effect one-dimensional: they are points. The nodes of a vibrating membrane are two-dimensional: they are lines. Finally, membranes display two kinds of nodes: circles concentric with the circumference of the drumhead and diameters that bisect the circumference.

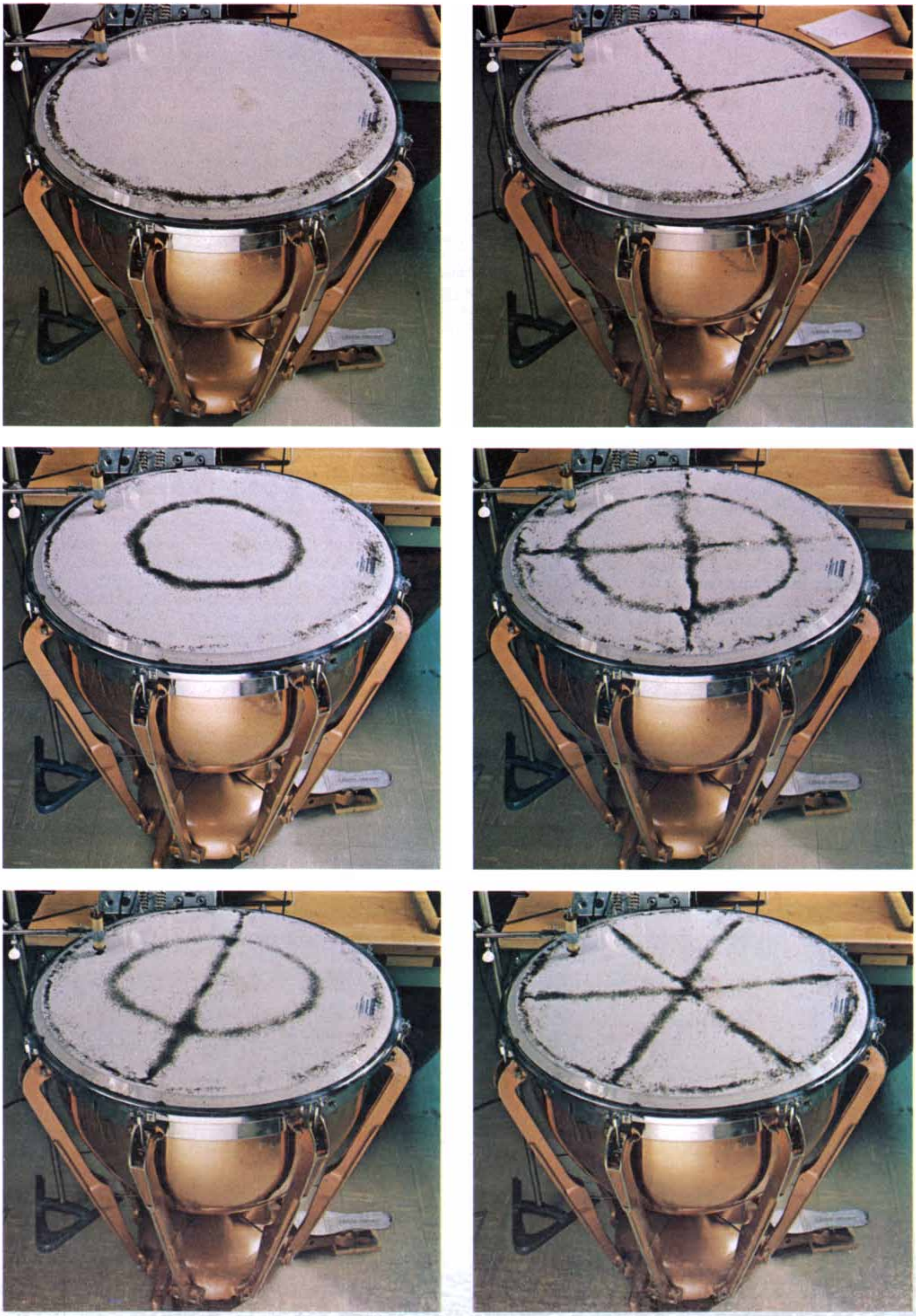
It is convenient to label the mode frequencies of a drumhead with pairs of digits. The first digit gives the number of diametric nodes in the mode frequency and the second gives the number of circular nodes. For example, when the oscillation of the membrane lacks any nodes except one at the circumference of the drumhead, the mode is labeled (01). If this mode is taken to be the fundamental mode of an ideal membrane, it is assigned a value of 1. The second mode, the one with the next-highest frequency, has one circular node and one diametric node; it is therefore labeled (11). In an ideal membrane the frequency is 1.59 times that of the fundamental mode. The third mode, with one circular node and two diametric nodes at right angles to each other, is labeled (21); its frequency is 2.14. The fourth mode, (02), consists of two circular nodes; its frequency is 2.30.

The nodal lines of course reflect the overall motion of the vibrating membrane. In mode (01) the entire membrane moves up and down. In mode (11), where the membrane is bisected by a diametric node, the two halves of the membrane move in opposite directions. In mode (21) opposing quarters of the membrane move in opposite directions; in mode (02) the inner circle of the membrane and the outer circle move in opposite directions. As the number of nodes is multiplied and the mode frequency rises the vibratory motions become increasingly complex.

The fundamental mode frequency of a membrane is determined by a number of factors. For example, doubling the tension of the membrane raises its frequency by half an octave. The diameter of the membrane also affects frequency, but inversely. For example, the fundamental frequency of a membrane 20 inches in diameter is 60 percent higher than that of a membrane 32 inches in diameter.

More than a century ago the German polymath Hermann von Helmholtz recognized that a musical tone conveying a clear sense of pitch must have several strong harmonic overtones, that is, it must have not only a fundamental frequency but also overtones with frequencies related to the fundamental in whole-number ratios. Writing on the sensations of tone, he pointed out that tones with a moderately loud series of harmonics up to the sixth mode sound rich and musical. As I have mentioned, however, the successive vibrational modes of an ideal membrane are not in the ratios of whole numbers. The ratios are 1 : 1.59 : 2.14 : 2.30 : 2.65 : 2.92,

KETTLEDRUM HEAD, sprinkled with powder, displays six of its many modes of vibration in the form of Chladni patterns. The powder collects at nodes, areas of the drumhead where the vibrations are weakest. By convention the mode frequencies are assigned pairs of digits; the first digit gives the number of diametric nodes and the second the number of circular nodes. Here, from top to bottom at the left, the modes are (01), that is, one circumferential node but no diametric node; (02), two circular nodes but no diametric node, and (12), the same two circular nodes and one diametric node. From top to bottom at the right the modes are (21), two diametric nodes but only the circumferential circular node; (22), two circular nodes and two diametric nodes, and (31), one circular node and three diametric nodes. The arm almost touching the membrane at the upper left in each photograph causes a tiny magnet fixed to the drumhead to vibrate at preselected frequencies. The patterns are named for the German physicist Ernst Chladni (1756–1827), who studied the vibrations of plates and membranes in this way.



and the vibrations of such a membrane could scarcely convey a clear sense of pitch.

This observation brings us back from the ideal to the real, because even if an ideal membrane would not convey a clear sense of pitch, a kettledrum does. If this were not so, the kettledrum would not be the most important drum in the orchestra. Kettledrums can also be tuned over a range of more than an octave, adjusting the tension of the drumhead by means of six to eight screws around the circumference of the kettle. In addition to the tensioning screws most modern kettledrums have a pedal-operated tensioning mechanism that enables the timpanist to vary the tension of the drumhead over a rather wide range above the tension already established, giving him a tuning range greater than a musical sixth. The timpanist in a symphony orchestra typically plays three to five kettledrums, but more instruments and more timpanists are not uncommon. The high-water mark is probably Berlioz' ***Grand-Messe des Morts,*** a piece that calls for 16 kettledrums and 10 timpanists.

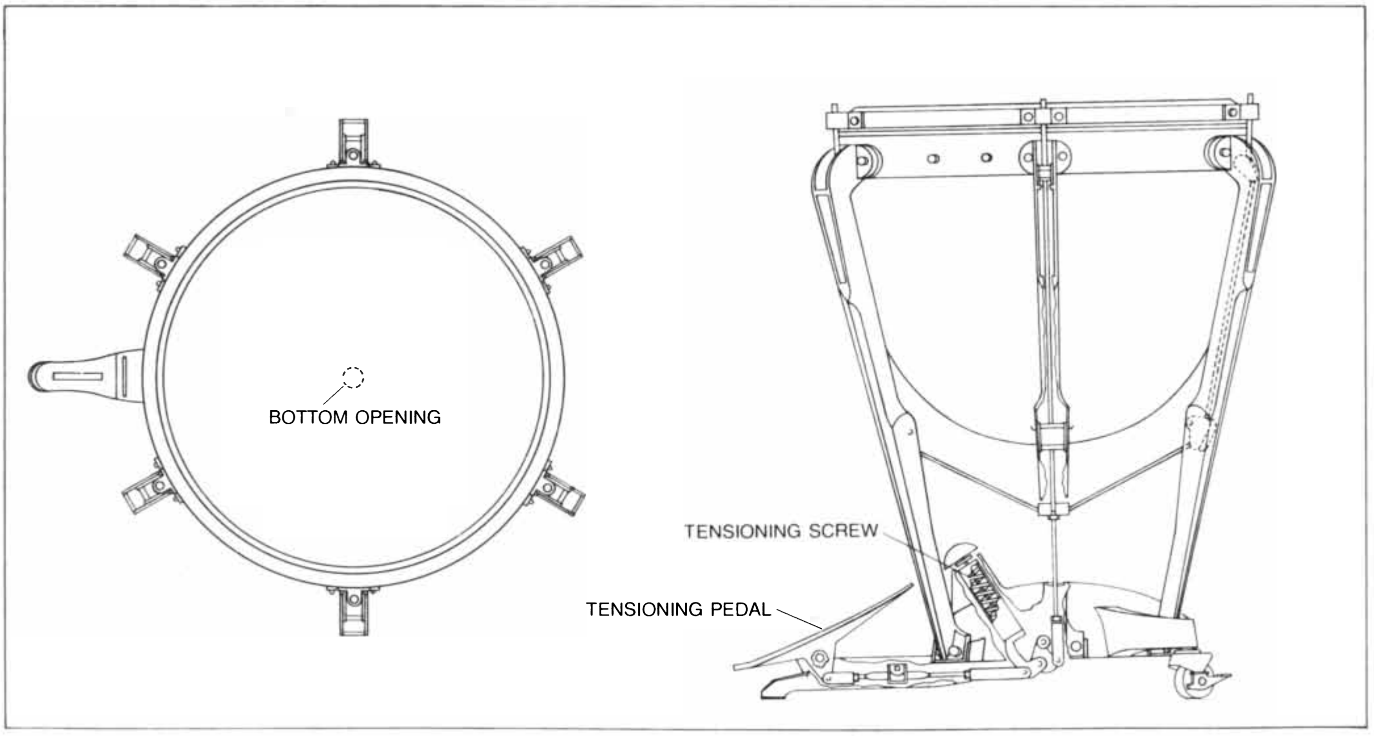
A carefully tuned kettledrum, correctly struck, sounds a strong principal tone and two or more harmonics. Late in the 19th century the British physicist Lord Rayleigh recognized that the principal tone was generated by the vibrations of the (11) mode. He further identified three successive harmonics: musically about a perfect fifth, a major seventh and an octave above the principal tone. These he ascribed respectively to vibrations in the (21), (31) and (12) modes. Their ascending frequencies, taking the (11) mode as the fundamental, are 1.50, 1.88 and 2.00 times the fundamental. In an ideal membrane the frequencies would be 1.34, 1.66 and 1.83.

Considering the fact that sophisticated acoustical apparatus did not exist in Rayleigh's day, his results are remarkable. In our acoustics laboratory at Northern Illinois University my colleagues and I have measured the various modes of kettledrums with modern instruments. These studies have led us farther along the path than Rayleigh got, but still in the same direction he pioneered a century ago. For example, we have found that the (11), (21) and (31) modes of the kettledrum have frequencies that are nearly in the ratios 1 : 1.5 : 2, and that the (41) and (51) modes typically have frequencies that are 2.44 and 2.90 times the frequency of the fundamental mode (11). These two values are within about half a semitone of 2.5 and 3. Thus a family of modes having respectively one, two, three, four and five diametric nodes radiate prominent tones with frequency ratios nearly in the whole-number sequence 2 : 3 : 4 : 5 : 6. It is these harmonics that give the kettledrum its strong sense of pitch.

How are the inharmonic modes of the ideal membrane coaxed into a harmonic relation? The main agent is the effect of air-mass loading. The imaginary ideal membrane vibrates in an ideal vacuum, and the drumhead of a real kettledrum vibrates at the bottom of an ocean of air. The mass of the air that sloshes back and forth lowers the frequencies of the modes of vibration. This downward shift, which is particularly important in the (11) mode, is mainly responsible for establishing the harmonic relations among kettledrum modes.

Two other effects may be said to "fine-tune" the drumhead frequencies, because their role is minor compared with the effect of air-mass loading. The first effect is that of the air enclosed in the kettle. The enclosed air, of course, has resonances of its own, and these have at least the potential of interacting with modes of the drumhead that have similar frequencies.

The second effect is that of the "stiffness" of the membrane. Like the stiffness of a string, that is, the string's resistance to bending, it tends to raise the frequencies of the higher harmonics. Stiffness in a "one-dimensional" string, however, differs from stiffness in a "two-dimensional" membrane, which may be characterized as resistance to shear. Like a sheet of paper, a membrane offers little resistance to the kind of distortion involved in bending it along a line. It offers strong resistance, however, to the kind of distortion that would be involved in wrapping it around a ball without wrinkling it. Neverthe-



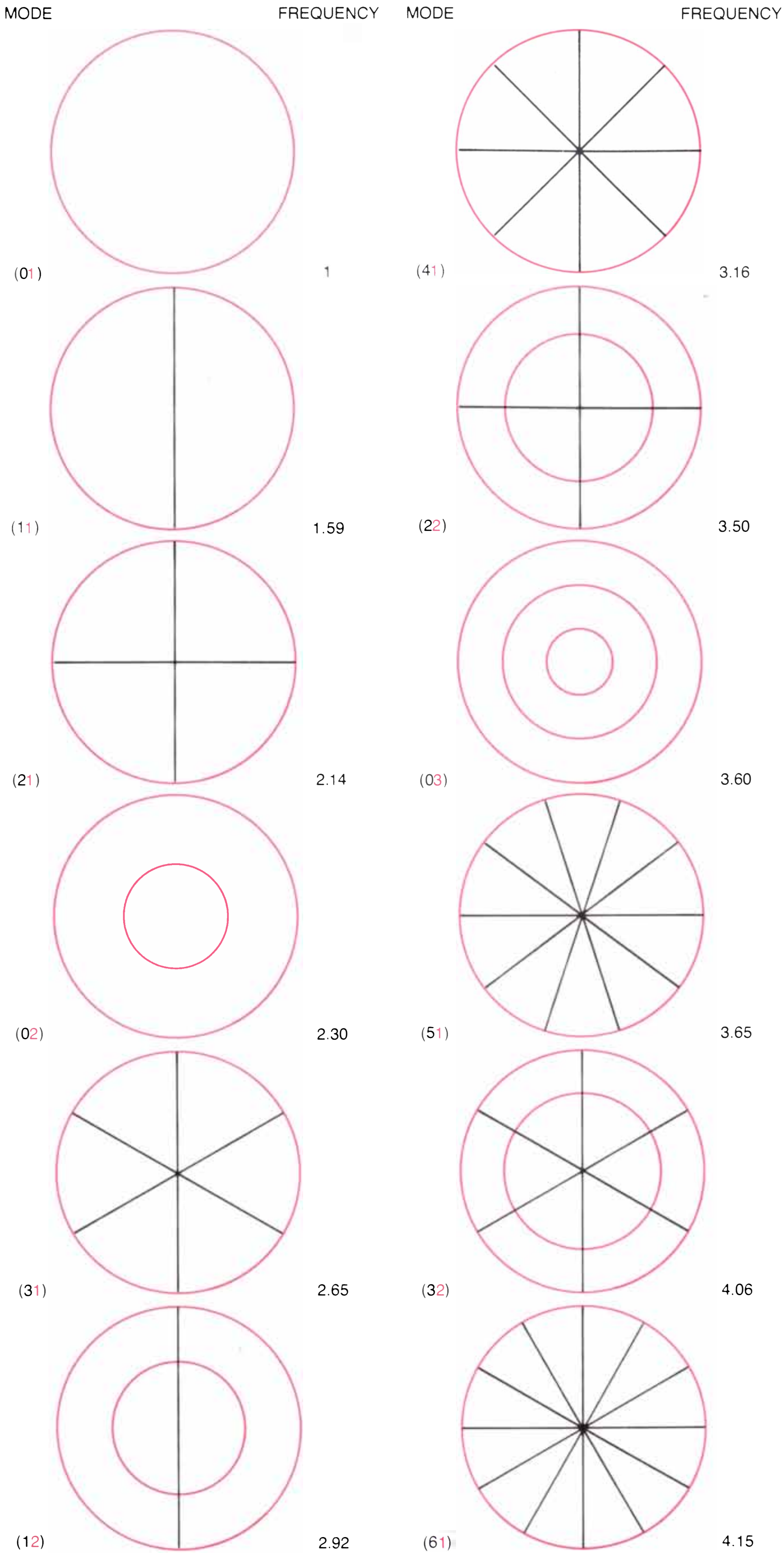
KETTLEDRUM is shown schematically in plan and elevation. Visible in the plan view are the six screws around the circumference of the kettle that make it possible to tune the drum over a range of about one octave. The broken circle indicates the opening at the bottom of the kettle (which is not present in all kettledrums). The partial cutaway view shows the pedal-controlled tensioning mechanism that enables the timpanist to alter the drum's tuning in the course of the performance. This instrument is made by Ludwig Industries of Chicago.

less, membrane stiffness or resistance to shear has the same effect resistance to bending has in a string: the frequencies of the higher harmonics rise. This is not to say that the resonance of the air inside the kettle and the drumhead's resistance to shear have no influence on kettledrum performance. Although they merely fine-tune the harmonic modes, they have a considerable effect on another component of kettledrum acoustics: the rate at which the sound of the drum decays after the drumhead is struck.

A number of conclusions about air loading can be arrived at by comparing the recorded frequencies of kettledrum modes when the instrument is in its usual condition with the frequencies recorded when the kettle has been removed. We did this in our laboratory, using a drum with a head 26 inches in diameter and comparing the ratios of the harmonic modes with the principal mode (11). Both with and without the kettle the frequencies of the (21) and the (31) modes were close to the ones measured by Rayleigh: a perfect fifth and an octave above the fundamental.

A further conclusion received support from a common acoustical model: a piston vibrating in the middle of a large baffle. Although this model is unrealistic with respect to a real kettledrum, it did show that as the frequency of vibration rises the air loading decreases markedly. A mathematical technique applied to air-loading calculations by Richard S. Christian and Arnold Tubis of Purdue University yielded results that are in even better agreement with our frequency measurements with and without the kettle than the results obtained from the simple piston model. In brief, air loading causes a considerable decrease in the frequency of the lower-frequency modes but has only a small influence on the frequency of the higher-frequency modes.

The spectrum of the sound radiated from a struck kettledrum depends on a number of factors: the point at which the drumhead is struck, the shape and the hardness of the felt-covered beater, the strength and nature of the stroke and even the position of the instrument and player in the room. The importance of striking the kettledrum membrane at the "normal" place in order to generate the desired harmonics is readily demonstrated with sound spectra: the normal place is about a quarter of the way from the edge of the drumhead to the center. If the drum is struck in the center, both the fundamental mode (01) and the immediately succeeding circular modes, (02) and (03), appear much stronger in the spectrum than the harmonics. The circular modes damp out rather quickly, however, and so do not account for much of the drum's



NODAL LINES of the 12 lowest modes of an ideal membrane are diagrammed in order of ascending frequency. The circular nodes and their mode numbers are in color; the diametric nodes and their mode numbers are in black. Frequency of vibration, calculated as a multiple of the "basic" (01) mode, appears to the right of each diagram. The sequence is not harmonic.

sound. Indeed, the membrane's response to being struck in the center is a dull thump.

A kettledrum 26 inches in diameter served for our sound-spectrum recordings. If the heavily damped (01) fundamental is ignored, the principal tone and the overtones generated by a normal stroke were very nearly in the ratios 1 : 1.5 : 2 : 2.5 : 3, a harmonic series built on a nonexistent fundamental an octave below the real one. This harmonic relation gives the sound of the kettledrum a strong sense of pitch and a pleasing timbre. Measurements on kettledrums of different sizes yielded similar results.

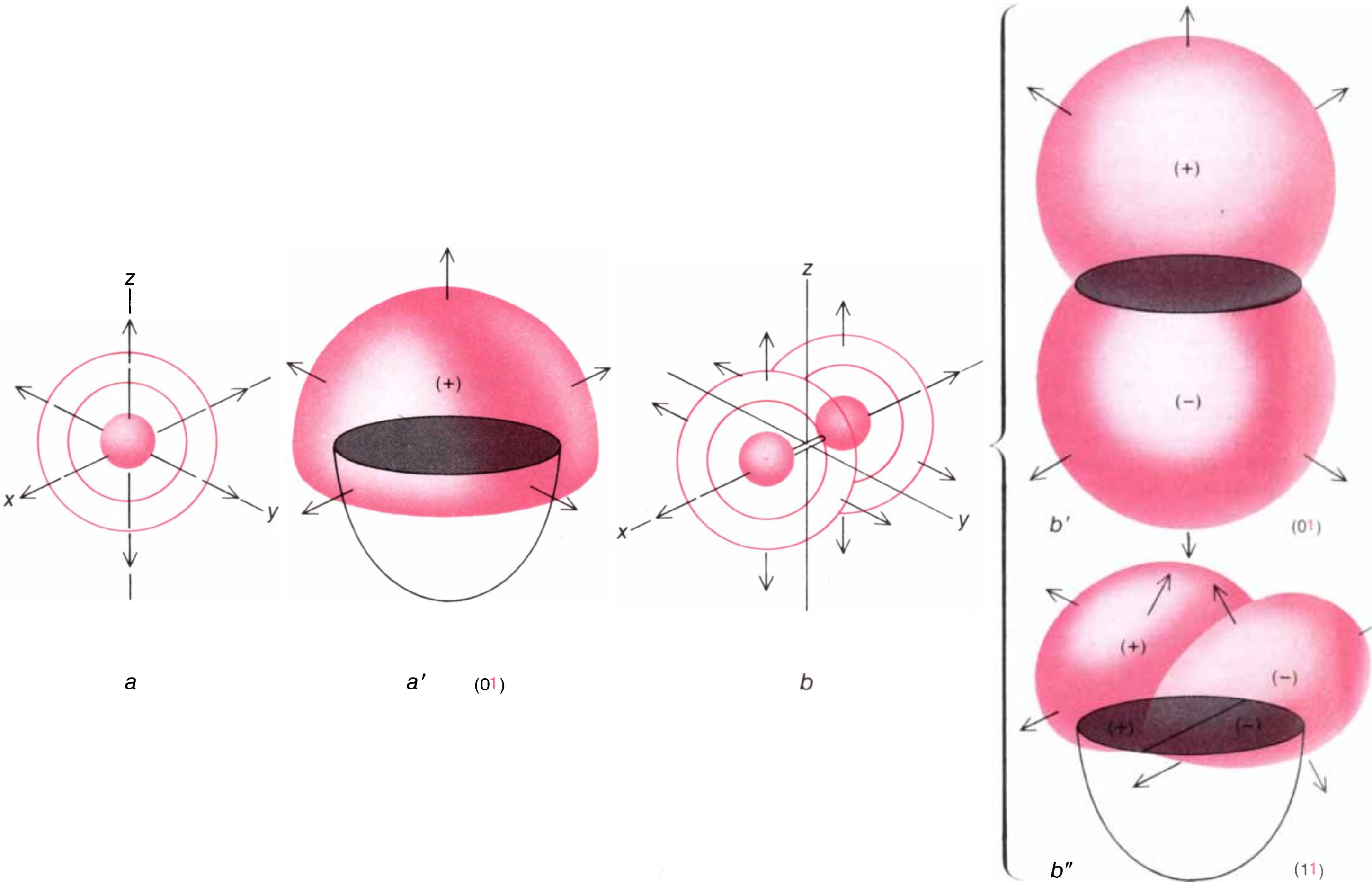
One of our observations remains unexplained. Why should the pitch of the kettledrums correspond to the pitch of the principal tone rather than to the missing fundamental of the harmonic series? Apparently the reason is that compared with the principal tone the strengths and durations of the harmonics are insufficient to establish the entire harmonic series. Some timpanists report, however, that a gentle stroke at the proper spot with a soft beater will produce a rather indistinct sound an octave lower than the kettledrum's nominal pitch.

The radiation of sound from a complex source is often described in terms of simple models consisting of oscillating spheres. The simplest of these models is a single sphere that alternately expands and contracts, sending out spherical sound waves. Such a single-sphere model is called a monopole source. The total sound power radiated from a monopole source is proportional to the square of the frequency at which it oscillates, and the sound radiates equally in all directions.

A model consisting of two oscillating spheres that expand and contract in opposite phase, separated by a given distance, is a dipole sound source. The radiation from a dipole varies greatly according to angle, being greatest along the perpendicular axis and decreasing to zero at 90 degrees from the perpendicular. The sound power of a dipole is proportional to the fourth power of the frequency; at low frequencies a dipole source radiates considerably less sound than a comparable monopole source. Two dipole sources oscillating in opposite phase constitute a quadrupole sound source. Its sound power is proportional to the sixth power of its frequency and so a quadrupole is an even less efficient radiator of sound at low frequencies than a dipole.

Consider these three models in relation to the radiation of sound by a kettledrum. When the drum is vibrating in its lowest mode, (01), all parts of the membrane move in phase, that is, they all move up at the same time and down at the same time. If the drumhead is attached to its kettle, which acts as a baffle, the sound radiating from the bottom surface cannot interact with the sound radiating from the top surface and so the drum is in effect a monopole source.

If the drumhead is not attached to the kettle, the sound radiation from the top surface of the membrane interferes with the sound radiation from the bottom surface and no sound is radiated in directions lying in the plane of the mem-



THREE SIMPLE MODELS of sound radiation are compared with their kettledrum equivalents; two are represented as drumheads alone and also as complete instruments. The first model (*a*) is a monopole source; it rapidly expands and contracts, sending spherical sound waves equally along *x*, *y* and *z* axes. The monopole is acoustically equivalent to a kettledrum (*a′*) vibrating in the (01) mode. Sound radiating from the bottom of the membrane cannot interact with the sound from the top, and so the drum is a monopole source. The second model is a dipole (*b*): the air supply in two adjacent balloons is pumped back and forth. As one source expands the other contracts; sound radiation is maximal along the dipole axis (*x*) and minimal along the axes at right angles (*y*, *z*). An unattached drumhead (*b′*) vibrating in the (01) mode is acoustically equivalent to a dipole source. The radiation from its top surface interferes with the radiation from its bottom surface, and no sound radiates in the directions along the plane of the membrane (*dipole* y *and* z *axes*). An intact kettledrum brane. Thus the drumhead is a dipole. So too is a drumhead with its kettle when it is vibrating in the (11) mode and no sound is radiated in a direction perpendicular to the membrane. If a drumhead without a kettle vibrates in its (11) mode, radiation from the dipole source on the top surface interferes with radiation from the dipole source on the bottom surface and the drumhead becomes a quadrupole source. A drumhead with its kettle, vibrating in the (21) mode, is also a quadrupole source.

These simple models of sound radiation tell several things about the acoustical performance of the kettledrum. For example, the sound radiation of each mode has a different spatial pattern. In the concert hall the expectation is that the sound of all the patterns is mixed more or less uniformly by the many reflections from the walls and other surfaces. That is generally the case, provided the listener is several wavelengths away from the instrument. The timpanist or anyone else close to the kettledrum, however, is likely to hear a sound quite different from the one the audience hears.

The intensity of the sound radiation affects another acoustical characteristic of the kettledrum: the rate at which the sound decays. As we have seen, when a kettledrum sound damps out quickly (proof that it is being radiated efficiently), it is likely to be heard as a dull, unmusical thump. Modes that radiate as quadrupoles or higher-order multipoles radiate less efficiently than monopoles or dipoles and can therefore be expected to decay more slowly and more musically. This slow-decay effect is to a considerable extent offset, however, by the fact that the efficiency of multipole radiation increases rapidly as the frequency of the sound rises. For example, one can predict that as the tension of the drumhead (and therefore the frequency of its vibration) is increased the decay time will decrease. Measurements made by Ronald Mills in our laboratory showed this prediction to be correct.

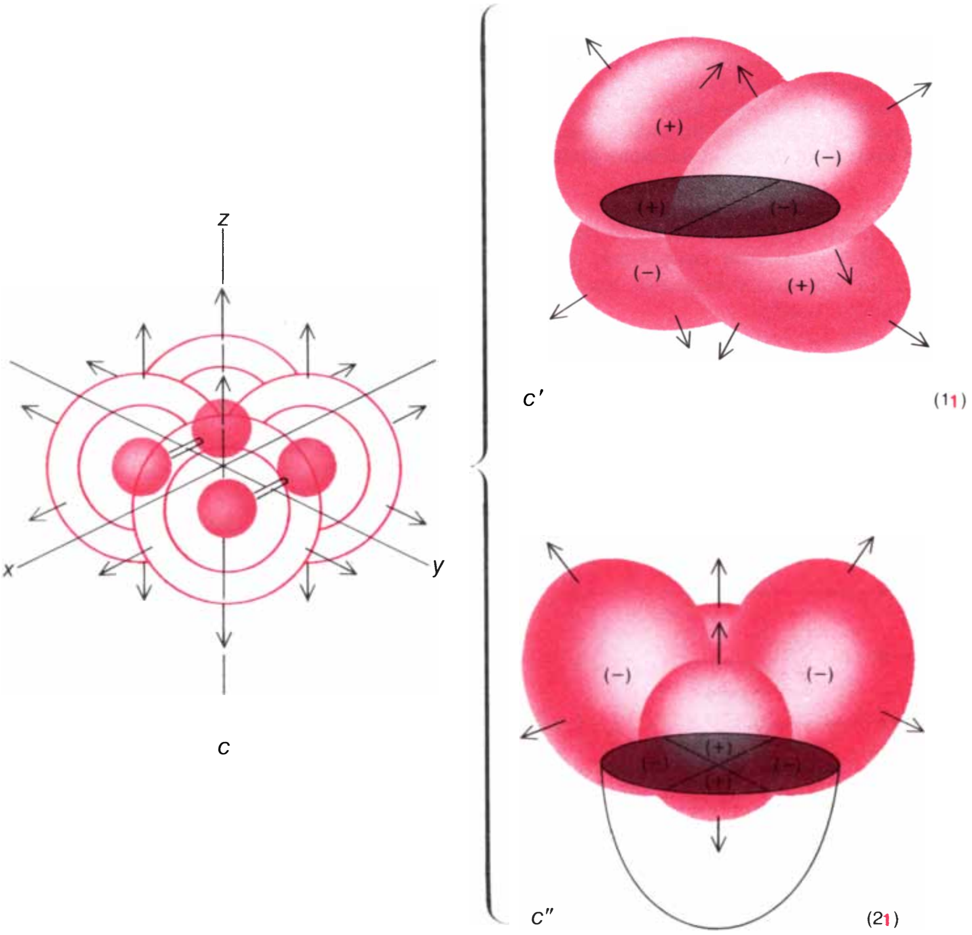
Of the various possible mechanisms responsible for the speed of damping (that is, the rate of energy loss from the vibrating membrane) the relative efficiency of sound radiation is paramount. Nevertheless, timpanists' own subjective judgments suggest that two other mechanisms may play a part. For example, most timpanists prefer calfskin drumheads to Mylar ones, a hint that there is a greater mechanical loss in the Mylar membrane. Many timpanists also think a drum with a copper kettle sounds different from one with a kettle made out of fiberglass or some other synthetic material, implying that a higher rate of energy loss in the nontraditional kettles may also increase the speed of damping.

What acoustical purpose is served by the kettle of the kettledrum, other than providing a baffle that isolates the bottom of the membrane from the top? One noticeable effect is that the presence of the kettle increases the frequency of the circular modes. The effect is strongest in the mode (01); the drumhead alone vibrates at a frequency of 82 hertz (cycles per second), and with the kettle in place the frequency is 127 hertz. The effect diminishes at higher mode frequencies but remains positive. For example, in the mode (02) the frequency of the drumhead alone is 241 hertz, compared with 252 hertz when the kettle is in place, and in the mode (03) the figures are respectively 407 and 418 hertz.

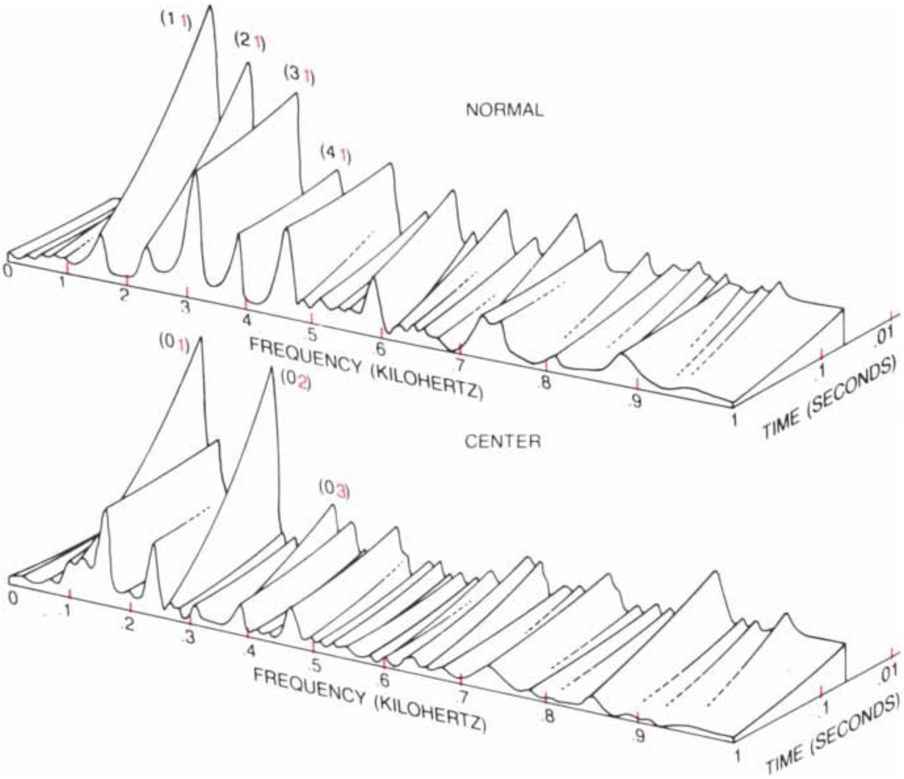
This increase in frequency comes about because in the circular modes the movement of the membrane alternately compresses and decompresses the air inside the kettle. The difference between the air pressure inside the kettle and the pressure outside acts as a restoring force of its own, over and above the restoring force inherent in the tension of the membrane. Call the effect air stiffness, similar to the action of the air enclosed in the airtight cabinet of an acoustic suspension speaker.

Just how much the air stiffness raises the frequency of a circular mode can be calculated. For a 26-inch kettle, which contains about .14 cubic meter of air, the frequency of the mode (01) is raised by about 40 percent by air stiffness if the membrane is at high tension and by about 80 percent if it is at low tension. The next mode, (02), is raised in frequency respectively by 2 percent and 6 percent; the (03) and (04) modes are raised by less than 1 percent.

How large should the kettle of a kettledrum be? Craig A. Anderson and I looked into the question by means of a simple experiment: reducing the air volume of the kettle by partially filling it with water. We then tested the effect the changing air volume had on the various diametric modes of vibration. Except for the principal mode, (11), the changes



(*b*″) vibrating in the (11) mode is also equivalent to a dipole source; no sounds are radiated in the directions perpendicular to the drumhead. The third model (*c*) is a quadrupole. The balloons at opposite ends of each dipole expand as the others contract; sound radiation is maximal along the *x* and *y* axes but minimal along the *z* axis. A solitary drumhead (*c*′) vibrating in the (11) mode is acoustically equivalent: vibrations from the top dipole surface interfere with vibrations from the bottom dipole surface. An intact kettledrum (*c*″) vibrating in the (21) mode is also equivalent to a quadrupole. Monopole sound output is proportional to the square of its frequency, dipole output to the fourth power and quadrupole output to the sixth power.



TWO SPECTRAL DIAGRAMS, constructed from spectra recorded soon after the instrument was struck and others recorded later, show the different mode peaks and the different decay times when a 26-inch kettledrum is struck at the "normal" point (*top*) and at the center of the drumhead (*bottom*). The normal stroke, emphasizing the related (11), (21), (31) and (41) modes, conveys a definite pitch, and the relatively protracted decay time allows the sound to "ring." The center stroke emphasizes the unharmonic modes (01), (02) and (03), all of which tend to decay rapidly. The sound that results is a kind of short-lived thump of indefinite pitch.

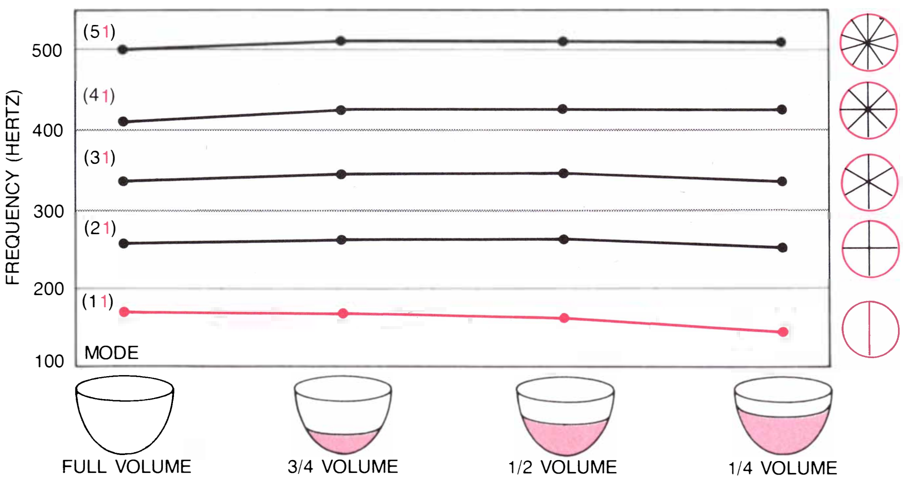
in the vibration pattern, as measured in hertz, were not particularly great even when the kettle was three-quarters filled with water. For the principal mode, however, the changes were substantial. When there was only a fourth of the normal amount of air in the kettle, the frequency of the (11) mode was reduced by 13 percent, from a normal 170 hertz to 148. This reduction completely destroyed the harmonic relations among the diametric modes.

Just as the kettledrum membrane exhibits a variety of modes, so does the mass of air inside the kettle. Anderson and I investigated these modes also, because in stringed instruments such as violins and guitars the vibrational modes of the confined air are known to have a noticeable effect on the modes of the wood top plate of the instrument. We wanted to learn if the same was true of the kettledrum. We determined seven air-mass modes and found that their frequencies were substantially higher than those of the corresponding membrane modes. For example, the membrane mode (11) has a frequency of 150 hertz and the equivalent air-mass mode has a frequency of 337 hertz. Similarly, the membrane mode (12) has a frequency of 314 hertz and the equivalent air-mass mode has one of 816 hertz. As a result of these large differences the vibrations of the kettledrum air mass interact only slightly with the vibrations of the drumhead, but the slight interactions do result in subtle changes in the sound of the drum.

Most kettledrums have a small vent at the bottom. With another simple experiment we were able to disprove two hypotheses about the effect of the vent on the performance of the instrument. One hypothesis was that the main cause of the strong damping of the fundamental (01) mode is viscous friction of the air at the venthole. The other hypothesis was that in the absence of the vent the enclosed air mass would be "stiffer," and so the presence of the vent prevents the rise in drumhead frequency that would result from the added stiffness.

All our experiment needed was a rubber stopper that would close the venthole. We found that stoppering the kettle had little or no effect on the decay time of the mode (01). The stopper actually lowered the modal frequency, rather than raising it, by a very small amount: .4 percent. For example, in one tuning the observed frequency of the mode (01), with an open vent, was $135.4 \pm .1$ hertz and the decay time was $.29 \pm .05$ second. With the vent stoppered the frequency value was lowered to $134.9 \pm .1$ hertz and the decay time remained the same. These findings are consistent with statements made by several experienced timpanists that they could hear no consistent difference between the sound of a kettledrum with its venthole open and the sound of one with its venthole closed.

The studies in our laboratory have added to the understanding of the acoustics of kettledrums, but the acoustical properties of most other drums have not yet been carefully investigated. A notable exception is the research in the 1930's regarding the modes of the Indian tabla, a small hand-struck drum, conducted by the noted physicist C. V. Raman. It is to be hoped that in future years the fascinating subject of the physics of drums in general will receive more attention.



CHANGES IN KETTLE VOLUME affect the overtones of the kettledrum slightly but significantly. When the air volume in the kettle was reduced first by a fourth and then by half, the frequencies of the four overtones were scarcely affected. When the volume was reduced to a fourth, however, frequency of the principal (11) mode was lowered both by the decreased volume and by the increased internal air loading. Its relation to the (21) mode was no longer a multiple of 1.50 but had stretched to 1.67, disrupting its harmonic relations with the overtones.