The Wonderful World of Linear Programming

Aaron N.K. Yip

Math 108, Fall 2025

It all started from a ....

Diet Problem

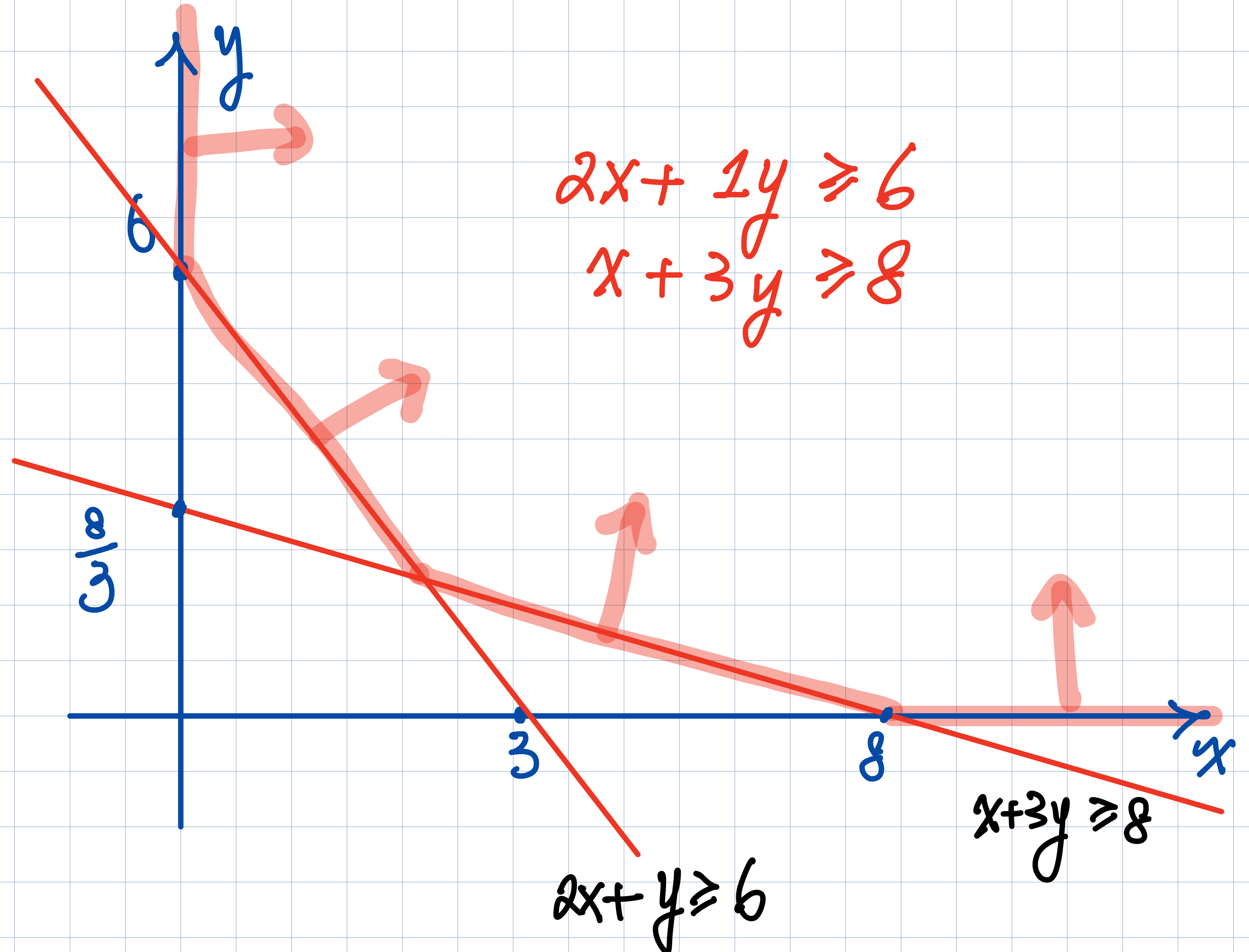
	Vitamin A	Vitamin C	Price
Carrot (g)	2 mg	1 mg	2 ¢
Cabbage (g)	1 mg	3 mg	3 ¢
min Daily Consumption	6 mg	8 mg	

	Vitamin A	Vitamin C	Price
Carrot (g) (x)	2 mg	1 mg	2¢
Cabbage (g) (y)	1 mg	3 mg	3¢
min Daily Consumption	6 mg	8 mg	

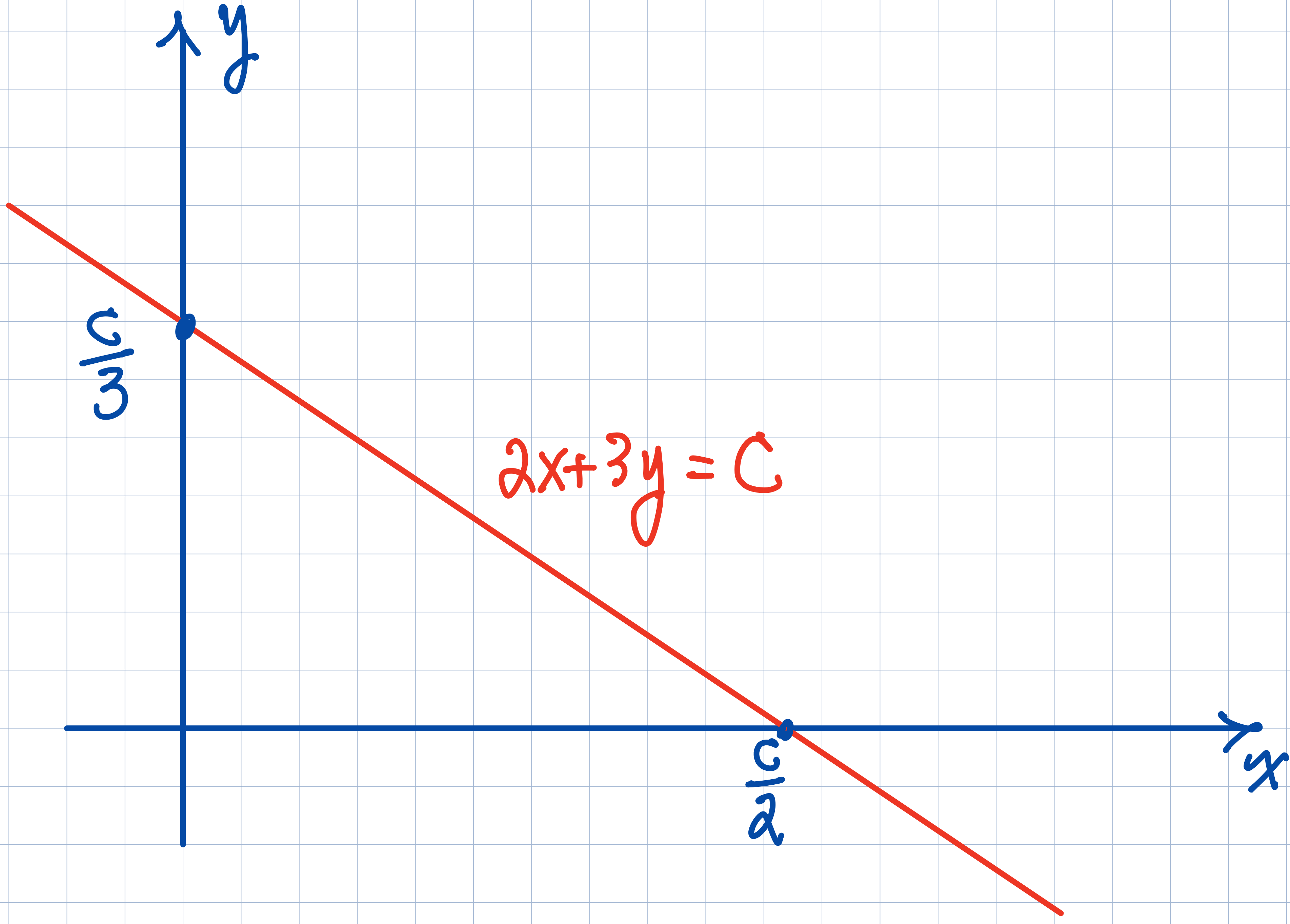
Cost : $2x + 3y$ ← minimize

Vitamin A : $2x + 1y \geq 6$

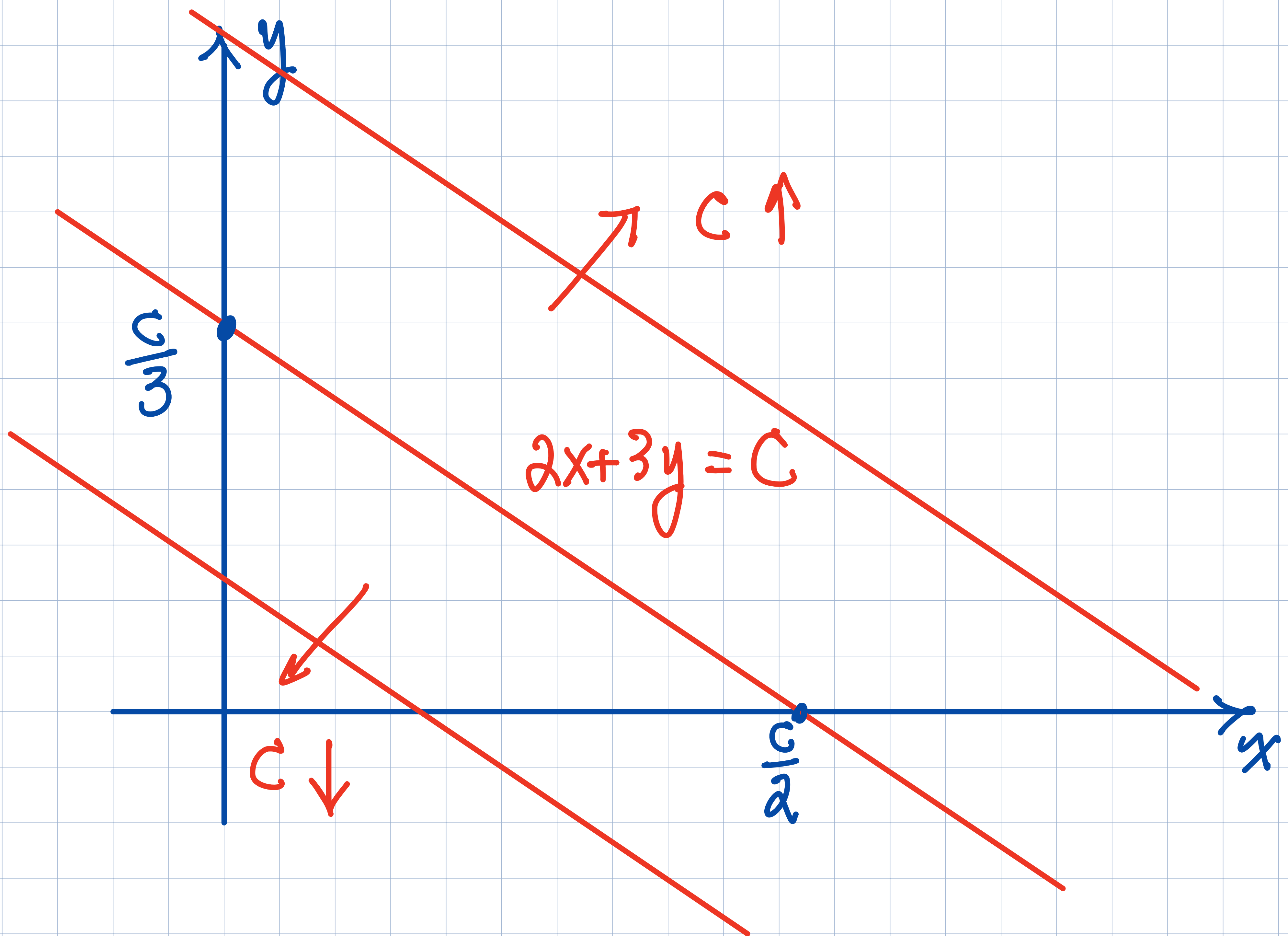
Vitamin C : $x + 3y \geq 8$



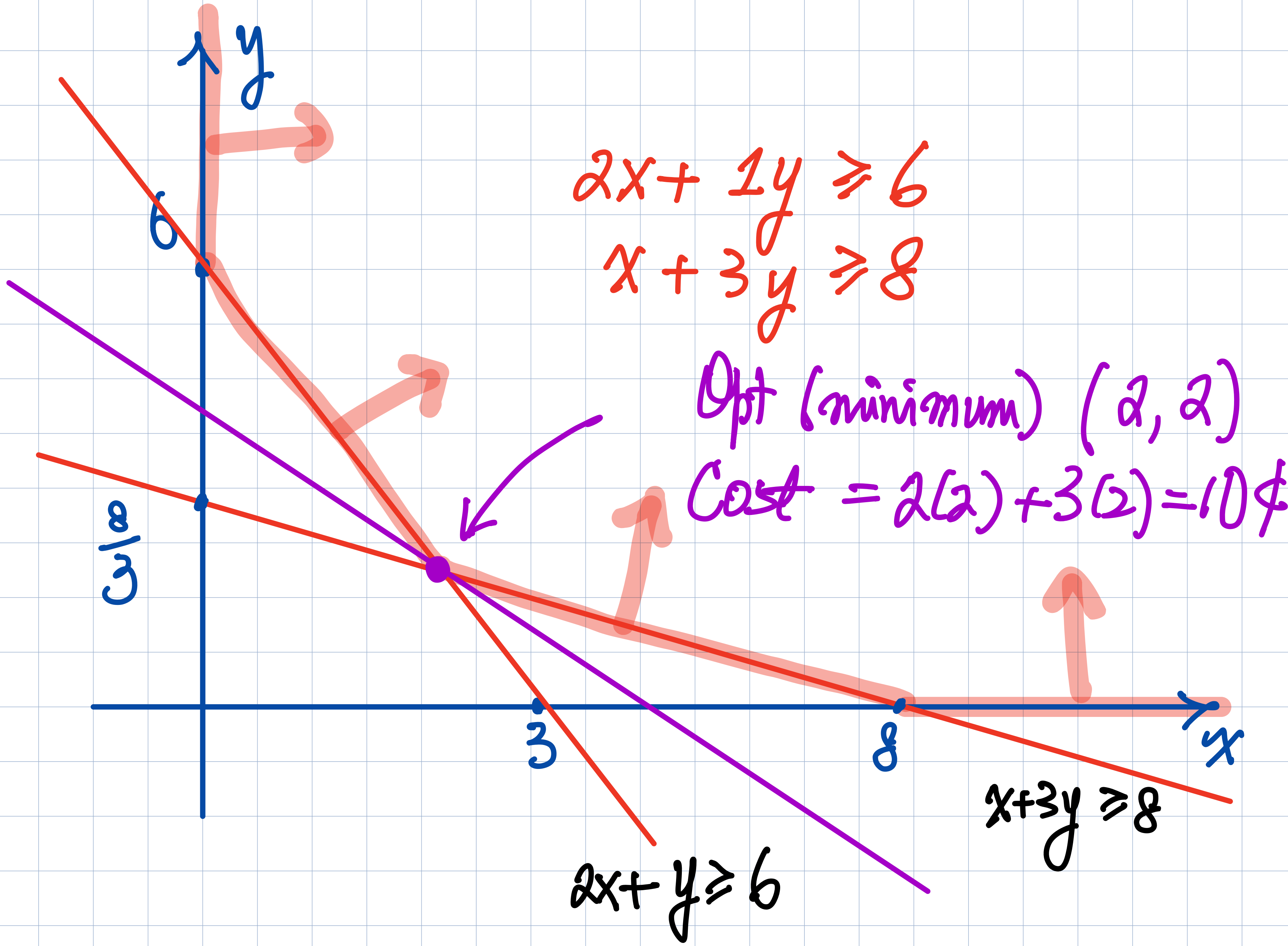
y
x
$2x + 1y \geq 6$
$x + 3y \geq 8$
6
$\frac{8}{3}$
3
8
$x+3y \geq 8$
$2x+y \geq 6$



y
$\frac{c}{3}$
$2x+3y=c$
$\frac{c}{2}$
x



y
x
c/3
c/2
2x+3y = C
C ↑
C ↓



y
2x+ 1y ≥ 6
x + 3y ≥ 8
6
Opt (minimum) (2, 2)
Cost = 2(2)+3(2)=10¢
8/3
3
8
x
x+3y ≥ 8
2x+ y ≥ 6

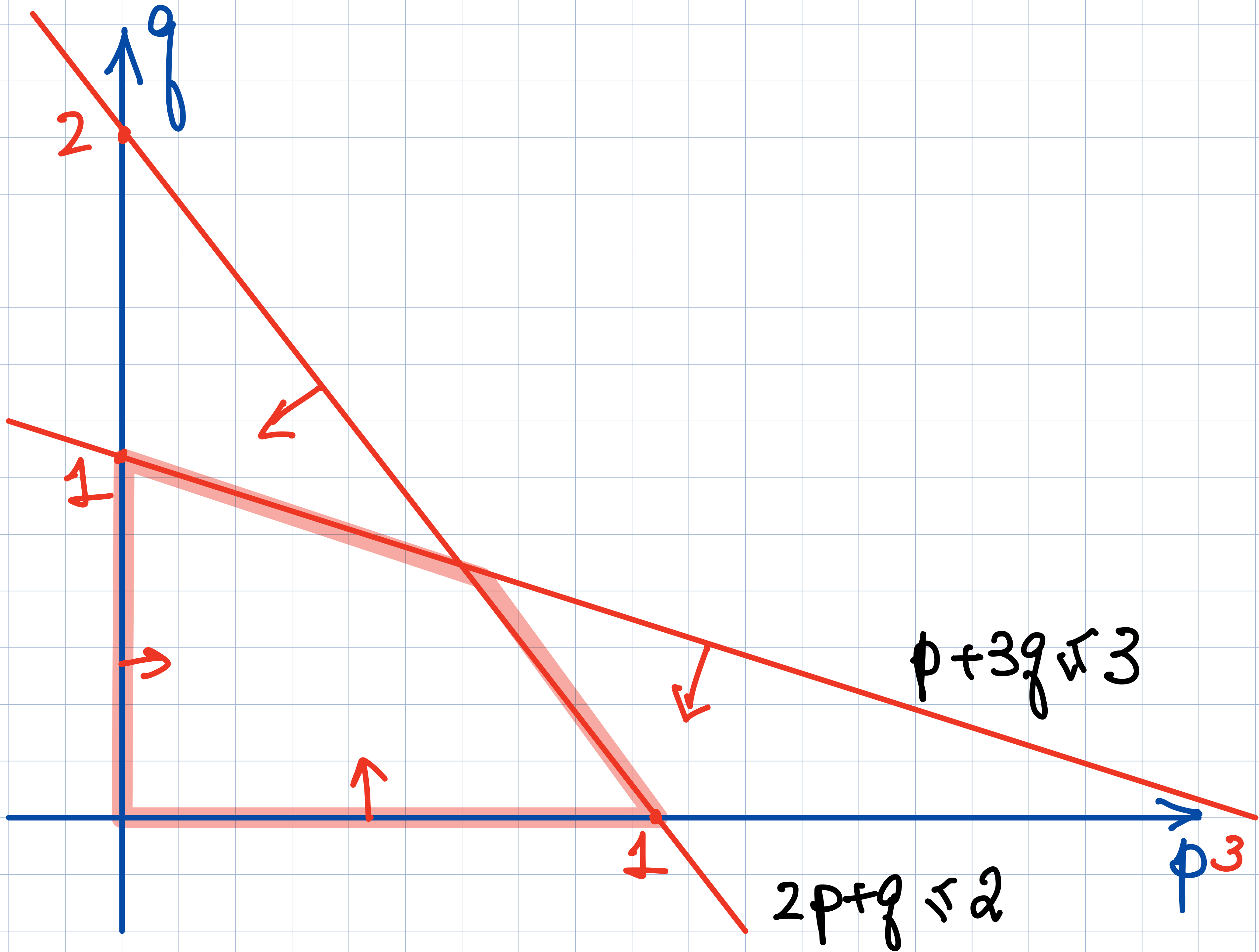
Bigpharma	(p) Vitamin A	(q) Vitamin C	Price
Carrot (g)	2 mg	1 mg	2 ¢
Cabbage (g)	1 mg	3 mg	3 ¢
min Daily Consumption	6 mg	8 mg	

Vitamin A Pill (p) Vitamin C Pill (q)

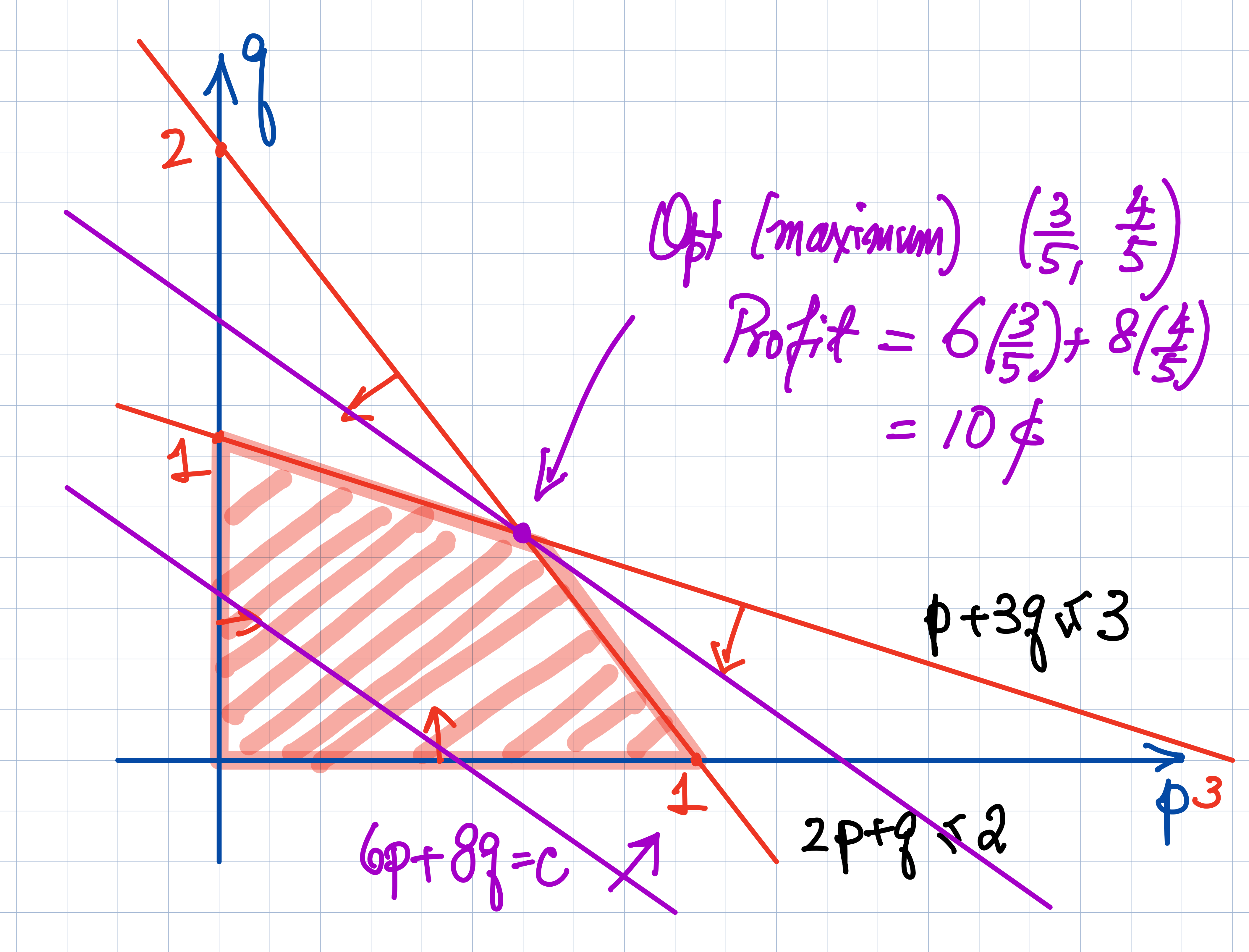
Profit (Daily) $6p + 8q$ ← maximize

Carrot Price $2p + q \leq 2$

Cabbage Price $p + 3q \leq 3$



q
2
1
1
p
3
p+3q ≤ 3
2p+q ≤ 2



q
2
1
1
p
3
Opt (maximum) $\left(\frac{3}{5}, \frac{4}{5}\right)$
Profit $= 6\left(\frac{3}{5}\right) + 8\left(\frac{4}{5}\right)$
$= 10$¢
$p + 3q \leq 3$
$2p + q \leq 2$
$6p + 8q = c$

The Cost of Subsistence
Author(s): George J. Stigler
Source: *Journal of Farm Economics*, May, 1945, Vol. 27, No. 2 (May, 1945), pp. 303-314

TABLE 1. DAILY ALLOWANCES OF NUTRIENTS FOR A MODERATELY ACTIVE MAN (weighing 154 pounds)*

Nutrient	Allowance
Calories	3,000 calories
Protein	70 grams
Calcium	.8 grams
Iron	12 milligrams
Vitamin A	5,000 International Units
Thiamine (B_1)	1.8 milligrams
Riboflavin (B_2 or G)	2.7 milligrams
Niacin (Nicotinic Acid)	18 milligrams
Ascorbic Acid (C)	75 milligrams

* National Research Council, *Recommended Dietary Allowances*, Reprint and Circular Series No. 115, January, 1943.

TABLE A. NUTRITIVE VALUES OF COMMON FOODS PER DOLLAR OF EXPENDITURE, AUGUST 15, 1939

Commodity	Unit	Price Aug. 15, 1939 (cents)	Edible Weight per $1.00 (grams)	Calories (1,000)	Protein (grams)	Calcium (grams)	Iron (mg.)	Vitamin A (1,000 I.U.)	Thiamine (mg.)	Riboflavin (mg.)	Niacin (mg.)	Ascorbic Acid (mg.)
**1. Wheat Flour (Enriched)	10 lb.	36.0	12,600	44.7	1,411	2.0	365		55.4	33.3	441	
2. Macaroni	1 lb.	14.1	3,217	11.6	418	.7	54		3.2	1.9	68	
3. Wheat Cereal (Enriched)	28 oz.	24.2	3,280	11.8	377	14.4	175		14.4	8.8	114	
4. Corn Flakes	8 oz.	7.1	3,194	11.4	252	.1	56		13.5	2.3	68	
5. Corn Meal	1 lb.	4.6	9,861	36.0	897	1.7	99	30.9	17.4	7.9	106	
6. Hominy Grits	24 oz.	8.5	8,005	28.6	680	.8	80		10.6	1.6	110	
7. Rice	1 lb.	7.5	6,048	21.2	460	.6	41		2.0	4.8	60	
8. Rolled Oats	1 lb.	7.1	6,389	25.3	907	5.1	341		37.1	8.9	64	
9. White Bread (Enriched)	1 lb.	7.9	5,742	15.0	488	2.5	115		13.8	8.5	126	
10. Whole Wheat Bread	1 lb.	9.1	4,985	12.2	484	2.7	125		13.9	6.4	160	
11. Rye Bread	1 lb.	9.2	4,930	12.4	439	1.1	82		9.9	3.0	66	
12. Pound Cake	1 lb.	24.8	1,829	8.0	130	.4	31	18.9	2.8	3.0	17	
13. Soda Crackers	1 lb.	15.1	3,004	12.5	288	.5	50					
14. Milk	1 qt.	11.0	8,867	6.1	310	10.5	18	16.8	4.0	16.0	7	177
**15. Evaporated Milk (can)	14½ oz.	6.7	6,035	8.4	422	15.1	9	26.0	3.0	23.5	11	60
16. Butter	1 lb.	30.8	1,473	10.8	9	.2	3	44.2		.2	2	
*17. Oleomargarine	1 lb.	16.1	2,817	20.6	17	.6	6	55.8	.2			
18. Eggs	1 doz.	32.6	1,857	2.9	238	1.0	52	18.6	2.8	6.5	1	
**19. Cheese (Cheddar)	1 lb.	24.2	1,874	7.4	448	16.4	19	28.1	.8	10.3	4	
20. Cream	½ pt.	14.1	1,689	3.5	49	1.7	3	16.9	.6	2.5		17
21. Peanut Butter	1 lb.	17.9	2,534	15.7	661	1.0	48		9.6	8.1	471	
22. Mayonnaise	½ pt.	16.7	1,198	8.6	18	.2	8	2.7	.4	.5		
23. Crisco	1 lb.	20.3	2,234	20.1								
24. Lard	1 lb.	9.8	4,628	41.7				.2		.5	5	
25. Sirloin Steak	1 lb.	39.6	1,145*	2.9	166	.1	34	.2	2.1	2.9	69	
26. Round Steak	1 lb.	36.4	1,246*	2.2	214	.1	32	.4	2.5	2.4	87	
27. Rib Roast	1 lb.	29.2	1,553*	3.4	213	.1	33			2.0		
28. Chuck Roast	1 lb.	22.6	2,007*	3.6	309	.2	46	.4	1.0	4.0	120	
29. Plate	1 lb.	14.6	3,107*	8.5	404	.2	62		.9			
**30. Liver (Beef)	1 lb.	26.8	1,692*	2.2	333	.2	139	169.2	6.4	50.8	316	525
31. Leg of Lamb	1 lb.	27.6	1,643*	3.1	245	.1	20		2.8	3.9	86	
32. Lamb Chops (Rib)	1 lb.	36.6	1,239*	3.3	140	.1	15		1.7	2.7	54	
33. Pork Chops	1 lb.	30.7	1,477*	3.5	196	.2	30		17.4	2.7	60	
34. Pork Loin Roast	1 lb.	24.2	1,874*	4.4	249	.3	37		18.2	3.6	79	
35. Bacon	1 lb.	25.6	1,772*	10.4	152	.2	23		1.8	1.8	71	
36. Ham—smoked	1 lb.	27.4	1,655*	6.7	212	.2	31		9.9	3.3	50	
37. Salt Pork	1 lb.	16.0	2,835*	18.8	164	.1	26		1.4	1.8		
38. Roasting Chicken	1 lb.	30.3	1,497*	1.8	184	.1	30	.1	.9	1.8	68	46
39. Veal Cutlets	1 lb.	42.3	1,072*	1.7	156	.1	24		1.4	2.4	57	
40. Salmon, Pink (can)	16 oz.	13.0	3,489	5.8	705	6.8	45	3.5	1.0	4.9	209	
41. Apples	1 lb.	4.4	9,072	5.8	27	.5	36	7.3	3.6	2.7	5	544
42. Bananas	1 lb.	6.1	4,982	4.9	60	.4	30	17.4	2.5	3.5	28	498
43. Lemons	1 doz.	26.0	2,380	1.0	21	.5	14		.5		4	952
44. Oranges	1 doz.	30.9	4,439	2.2	40	1.1	18	11.1	3.6	1.3	10	1,998
*45. Green Beans	1 lb.	7.1	5,750	2.4	138	3.7	80	69.0	4.3	5.8	37	862
**46. Cabbage	1 lb.	3.7	8,949	2.6	125	4.0	36	7.2	9.0	4.5	26	5,369
47. Carrots	1 bunch	4.7	6,080	2.7	73	2.8	43	188.5	6.1	4.3	89	608
48. Celery	1 stalk	7.3	3,915	.9	51	3.0	23	.9	1.4	1.4	9	313
49. Lettuce	1 head	8.2	2,247	.4	27	1.1	22	112.4	1.8	3.4	11	449
*50. Onions	1 lb.	3.6	11,844	5.8	166	3.8	59	16.6	4.7	5.9	21	1,184

*51. Potatoes	15 lb.	34.0	16,810	14.3	336	1.8	118	6.7	29.4	7.1	198	2,522
**52. Spinach	1 lb.	8.1	4,592	1.1	106	—	138	918.4	5.7	13.8	33	2,755
**53. Sweet Potatoes	1 lb.	5.1	7,649	9.6	138	2.7	54	290.7	8.4	5.4	83	1,912
54. Peaches (can)	No. 2½	16.8	4,894	3.7	20	.4	10	21.5	.5	1.0	31	196
55. Pears (can)	No. 2½	20.4	4,030	3.0	8	.3	8	.8	.8	.8	5	81
56. Pineapple (can)	No. 2½	21.3	3,993	2.4	16	.4	8	2.0	2.8	.8	7	399
57. Asparagus (can)	No. 2	27.7	1,945	.4	33	.3	12	16.3	1.4	2.1	17	272
58. Green Beans (can)	No. 2	10.0	5,386	1.0	54	2.0	65	53.9	1.6	4.3	32	431
59. Pork and Beans (can)	16 oz.	7.1	6,389	7.5	364	4.0	134	3.5	8.3	7.7	56	
60. Corn (can)	No. 2	10.4	5,452	5.2	136	.2	16	12.0	1.6	2.7	42	218
61. Peas (can)	No. 2	13.8	4,109	2.3	136	.6	45	34.9	4.9	2.5	37	370
62. Tomatoes (can)	No. 2	8.6	6,263	1.3	63	.7	38	53.2	3.4	2.5	36	1,253
63. Tomato Soup (can)	10½ oz.	7.6	3,917	1.6	71	.6	43	57.9	3.5	2.4	67	862
*64. Peaches, Dried	1 lb.	15.7	2,889	8.5	87	1.7	173	86.8	1.2	4.3	55	57
*65. Prunes, Dried	1 lb.	9.0	4,284	12.8	99	2.5	154	85.7	3.9	4.3	65	257
66. Raisins, Dried	15 oz.	9.4	4,524	13.5	104	2.5	136	4.5	6.3	1.4	24	136
67. Peas, Dried	1 lb.	7.9	5,742	20.0	1,367	4.2	345	2.9	28.7	18.4	162	
**68. Lima Beans, Dried	1 lb.	8.9	5,097	17.4	1,055	3.7	459	5.1	26.9	38.2	93	
**69. Navy Beans, Dried	1 lb.	5.9	7,688	26.9	1,691	11.4	792		38.4	24.6	217	
70. Coffee	1 lb.	22.4	2,025	—	—	—	—		4.0	5.1	50	
71. Tea	¼ lb.	17.4	652	—	—	—	—			2.3	42	
72. Cocoa	8 oz.	8.6	2,637	8.7	237	3.0	72		2.0	11.9	40	
73. Chocolate	8 oz.	16.2	1,400	8.0	77	1.3	39		.9	3.4	14	
74. Sugar	10 lb.	51.7	8,773	34.9	—	—	—					
75. Corn Sirup	24 oz.	13.7	4,966	14.7	—	.5	74				5	
76. Molasses	18 oz.	13.6	3,752	9.0	—	10.3	244		1.9	7.5	146	
77. Strawberry Preserves	1 lb.	20.5	2,213	6.4	11	.4	7	.2	.2	.4	3	

* Quantities including inedible portions.

TABLE B. NUTRITIVE VALUES OF COMMON FOODS PER DOLLAR OF EXPENDITURE, AUGUST 15, 1944

Commodity	Price Aug. 15, 1944 (cents)	Calories (1,000)	Protein (grams)	Calcium (grams)	Iron (mg.)	Vitamin A (1,000 I.U.)	Thiamine (mg.)	Riboflavin (mg.)	Niacin (mg.)	Ascorbic Acid (mg.)
1. Wheat Flour	64.6	24.9	786	1.1	203		30.9	18.6	246	
3. Wheat Cereal	23.2	12.3	398	15.0	183		15.0	9.2	119	
5. Corn Meal	6.3	26.3	655	1.2	72	22.6	12.7	5.8	77	
8. Rolled Oats	9.9	18.1	651	3.7	245		26.6	6.4	46	
15. Evaporated Milk	10.0	5.6	283	10.1	6	17.4	2.0	15.7	7	40
46. Cabbage	4.9	2.0	94	3.0	27	5.4	6.8	3.4	20	4,054
51. Potatoes	80.1	6.1	143	.8	50	2.8	12.5	3.0	84	1,071
52. Spinach	11.6	.8	74	—	96	641.3	4.0	9.6	23	1,924
53. Sweet Potatoes	12.3	4.0	57	1.1	22	120.5	3.5	2.2	34	793
69. Navy Beans	10.8	14.7	924	6.2	433		21.0	13.4	119	
74. Sugar	67.0	26.9	—	—	—					
78. Pancake Flour[1]	12.2	16.0	479	18.1	46		3.7	1.9	41	
79. Beets[2]	7.3	2.2	85	1.1	70	132.3	2.9	6.3	29	895
80. Liver (Pork)[3]	21.9	2.7	408	.2	518	145.0	10.4	51.8	472	580

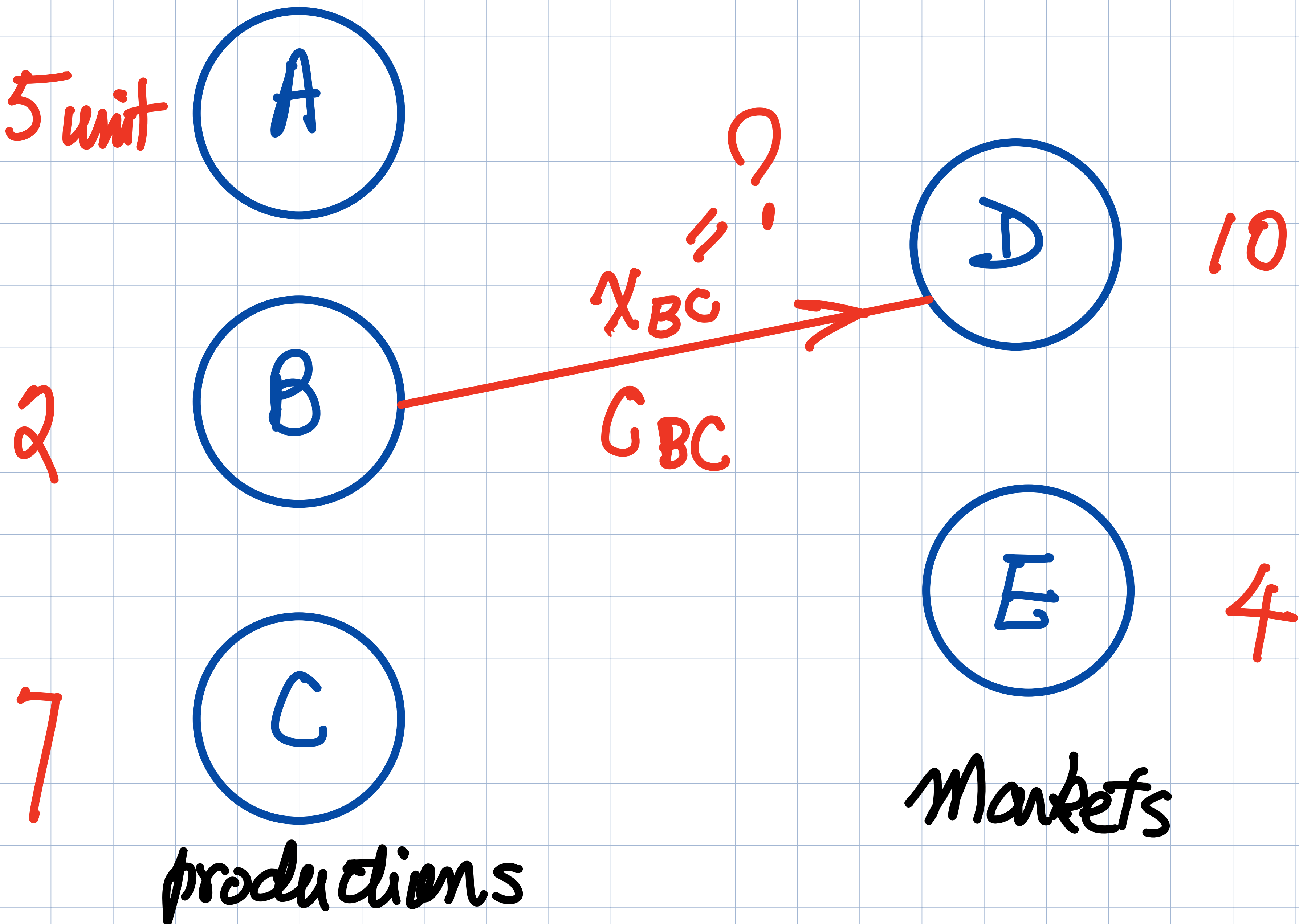
[1] Unit: 20 oz.; edible weight: 4,647 g. [2] Unit: 1 bunch; edible weight: 4,971 g. [3] Unit: 1 lb.; edible weight: 2,071 g.

The Cost of Subsistence
Author(s): George J. Stigler
Source: *Journal of Farm Economics*, May, 1945, Vol. 27, No. 2 (May, 1945), pp. 303-314

TABLE 2. MINIMUM COST ANNUAL DIETS, AUGUST 1939 AND 1944

Commodity	August 1939		August 1944	
	Quantity	Cost	Quantity	Cost
Wheat Flour	370 lb.	$13.33	535 lb.	$34.53
Evaporated Milk	57 cans	3.84	—	—
Cabbage	111 lb.	4.11	107 lb.	5.23
Spinach	23 lb.	1.85	13 lb.	1.56
Dried Navy Beans	285 lb.	16.80	—	—
Pancake Flour	—	—	134 lb.	13.08
Pork Liver	—	—	25 lb.	5.48
Total Cost		$39.93		$59.88

Transportation Problem



Transportation Problem

$$\text{Total cost} = \sum_{\substack{i=A,B,C \\ j=D,E}} C_{ij} x_{ij}$$

$$x_{AD} + x_{AE} = 5$$

$$x_{BD} + x_{BE} = 2$$

$$x_{CD} + x_{CE} = 7$$

$$x_{AD} + x_{BD} + x_{CD} = 10$$

$$x_{AE} + x_{BE} + x_{CE} = 4$$

$$(x_{ij} \geq 0)$$

Linear Programming

(and the world has changed....)

$$\min/\max \quad c^T x$$

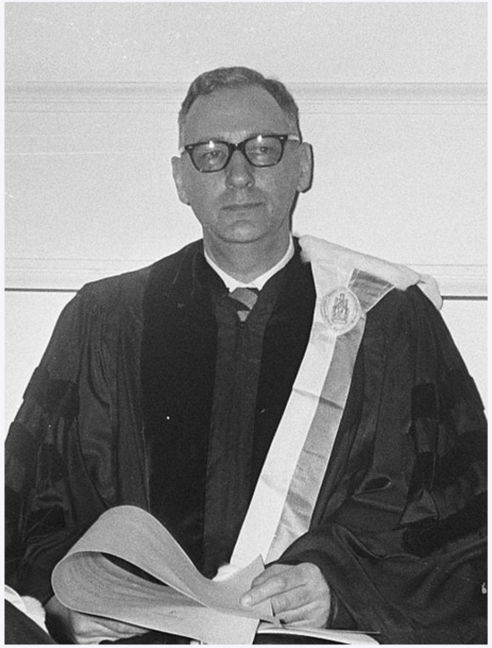
$$\text{subject to} \quad Ax \leq b$$

$$x \geq 0$$

Wikipedia, the free encyclopedia

Tjalling Charles Koopmans (August 28, 1910 – February 26, 1985) was a Dutch-American mathematician and economist.[1][2] He was the joint winner with Leonid Kantorovich of the 1975 Nobel Memorial Prize in Economic Sciences for his work on the theory of the optimum allocation of resources. Koopmans showed that on the basis of certain efficiency criteria, it is possible to make important deductions concerning optimum price systems.

Tjalling C. Koopmans



Koopmans in 1967

Born	August 28, 1910 's-Graveland, Netherlands

Biography [edit]

Koopmans was born in 's-Graveland, Netherlands. He began his university education at the Utrecht University at seventeen, specializing in mathematics. Three years later, in 1930, he switched to theoretical physics. In 1933, he met Jan Tinbergen, the winner of the 1969 Nobel Memorial Prize in Economics and moved to Amsterdam to study mathematical economics under him. In addition to mathematical economics, Koopmans extended his explorations to econometrics and statistics. In 1936, he graduated from Leiden University with a PhD, under the direction of Hendrik Kramers. The title of the thesis was "Linear regression analysis of economic time series".[3] He also worked for the Economic and Financial Organization of the League of Nations.[4]:28

CONCEPTS OF OPTIMALITY AND THEIR USES

Nobel Memorial Lecture, December 11, 1975

by
TJALLING C. KOOPMANS
Yale University, New Haven, Connecticut, USA

Leonid Kantorovich

63 languages

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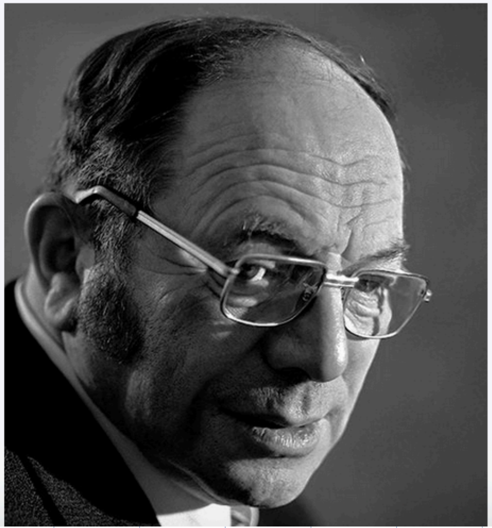
From Wikipedia, the free encyclopedia

Leonid Vitalyevich Kantorovich (Russian: Леонид Витальевич Канторович, IPA: [lʲɪɐˈnʲit vʲɪˈtalʲjɪvʲɪtɕ kəntɐˈrovʲɪtɕ] ⓘ; 19 January 1912 – 7 April 1986) was a Soviet mathematician and economist, known for his theory and development of techniques for the optimal allocation of resources. He is regarded as the founder of linear programming. He was the winner of the Stalin Prize in 1949 and the Nobel Memorial Prize in Economic Sciences in 1975.

Leonid Kantorovich
Леонид Канторович



Biography [edit]

Kantorovich was born on 19 January 1912, to a Russian Jewish family.[1] His father was a doctor practicing in Saint Petersburg.[2] In 1926, at the age of fourteen, he began his studies at Leningrad State University. He graduated from the Faculty of Mathematics and Mechanics in 1930, and began his graduate studies. In 1934, at the age of 22 years, he became a full professor. In 1935 he received his doctoral degree.[3]

Lecture to the memory of Alfred Nobel, December 11, 1975

Mathematics in Economics: Achievements, Difficulties, Perspectives

Leonid Kantorovich

63 languages

Article Talk Read Edit View history Tools

From Wikipedia, the free encyclopedia

Leonid Vitalyevich Kantorovich (Russian: Леонид Витальевич Канторович, IPA: [lʲɪɐˈnʲit vʲɪˈtalʲjɪvʲɪtɕ kəntɐˈrovʲɪtɕ]; 19 January 1912 – 7 April 1986) was a Soviet mathematician and economist, known for his theory and development of techniques for the optimal allocation of resources. He is regarded as the founder of linear programming. He was the winner of the Stalin Prize in 1949 and the Nobel Memorial Prize in Economic Sciences in 1975.

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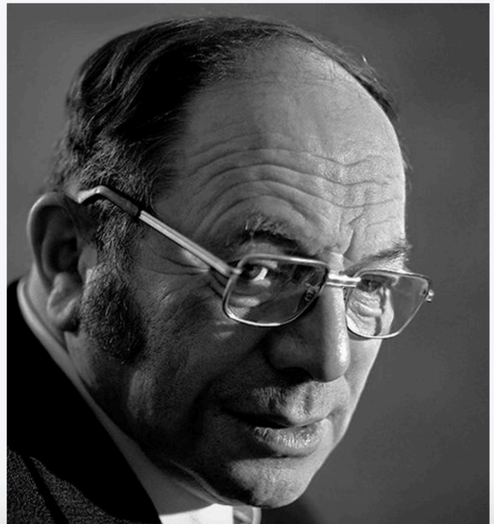

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At the beginning of the 1970s Kantorovich left Novosibirsk for Moscow where he was deeply engaged in economic analysis, not ceasing his efforts to influence the everyday economic practice and decision making in the national economy. His activities were mainly waste of time and stamina in view of the misunderstanding and hindrance of the governing retrogradists of this country. Cancer terminated his path in science on April 7, 1986. He was buried at Novodevichy Cemetery in Moscow.

(Math. & Econ. of Leonid Kantorovich, S.S. Kutateladze)

George Dantzig

37 languages

Article Talk Read Edit View history Tools

From Wikipedia, the free encyclopedia

George Bernard Dantzig (/ˈdæntsɪɡ/; November 8, 1914–May 13, 2005) was an American mathematical scientist who made contributions to industrial engineering, operations research, computer science, economics and statistics.

Dantzig is known for his development of the simplex algorithm,[1] an algorithm for solving linear programming problems, and for his other work with linear programming. In statistics, Dantzig solved two open problems in statistical theory, which he had mistaken for homework after arriving late to a lecture by Polish mathematician-statistician Jerzy Spława-Neyman.[2]

At his death, Dantzig was professor emeritus of Transportation Sciences and Professor of Operations Research and of Computer Science at Stanford University.

George Dantzig

Dantzig with President Gerald Ford in 1976

Born	George Bernard Dantzig November 8, 1914

An Interview with George B. Dantzig: The Father of Linear Programming
Author(s): Donald J. Albers, Constance Reid, George B. Dantzig
Source: *The College Mathematics Journal*, Vol. 17, No. 4 (Sep., 1986), pp. 293–314

George B. Dantzig (1914–2005)

Richard Cottle, Ellis Johnson, and Roger Wets

Notices of AMS

George Dantzig

37 languages

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George Bernard Dantzig (/ˈdæntsɪɡ/; November 8, 1914–May 13, 2005) was an American mathematical scientist who made contributions to industrial engineering, operations research, computer science, economics and statistics.

Dantzig is known for his development of the simplex algorithm,[1] an algorithm for solving linear programming problems, and for his other work with linear programming. In statistics, Dantzig solved two open problems in statistical theory, which he had mistaken for homework after arriving late to a lecture by Polish mathematician-statistician Jerzy Spława-Neyman.[2]

At his death, Dantzig was professor emeritus of Transportation Sciences and Professor of Operations Research and of Computer Science at Stanford University.

George Dantzig

Dantzig with President Gerald Ford in 1976

Born	George Bernard Dantzig November 8, 1914

LINEAR PROGRAMMING AND EXTENSIONS

by GEORGE B. DANTZIG

THE RAND CORPORATION

and

UNIVERSITY OF CALIFORNIA, BERKELEY

The Best of the 20th Century: Editors Name Top 10 Algorithms

By Barry A. Cipra

Algos is the Greek word for pain. *Algor* is Latin, to be cold. Neither is the root for *algorithm*, which stems instead from al-Khwarizmi, the name of the ninth-century Arab scholar whose book *al-jabr wa'l muqabalah* devolved into today's high school algebra textbooks. Al-Khwarizmi stressed the importance of methodical procedures for solving problems. Were he around today, he'd no doubt be impressed by the advances in his eponymous approach.

Some of the very best algorithms of the computer age are highlighted in the January/February 2000 issue of *Computing in Science & Engineering*, a joint publication of the American Institute of Physics and the IEEE Computer Society. Guest editors Jack Don-garra of the University of Tennessee and Oak Ridge National Laboratory and Fran-cis Sullivan of the Center for Comput-ing Sciences at the Institute for Defense Analyses put togeth-er a list they call the "Top Ten Algorithms of the Century."

"We tried to assemble the 10 al-gorithms with the greatest influence on the development and practice of science and engineering in the 20th century," Dongarra and Sullivan write. As with any top-10 list, their selections—and non-selections—are bound to be controversial, they acknowledge. When it comes to picking the algorithmic best, there seems to be no best algorithm.

Without further ado, here's the CiSE top-10 list, in chronological order. (Dates and names associated with the algorithms should be read as first-order approximations. Most algorithms take shape over time, with many contributors.)

1946: John von Neumann, Stan Ulam, and Nick Metropolis, all at the Los Alamos Scientific Laboratory, cook up the Metropolis algorithm, also known as the **Monte Carlo method**.

The Metropolis algorithm aims to obtain approximate solutions to numerical problems with unmanageably many degrees of freedom and to combinatorial problems of factorial size, by mimicking a random process. Given the digital computer's reputation for deterministic calculation, it's fitting that one of its earliest applications was the generation of random numbers.

In terms of widespread use, George Dantzig's simplex method is among the most successful algorithms of all time.

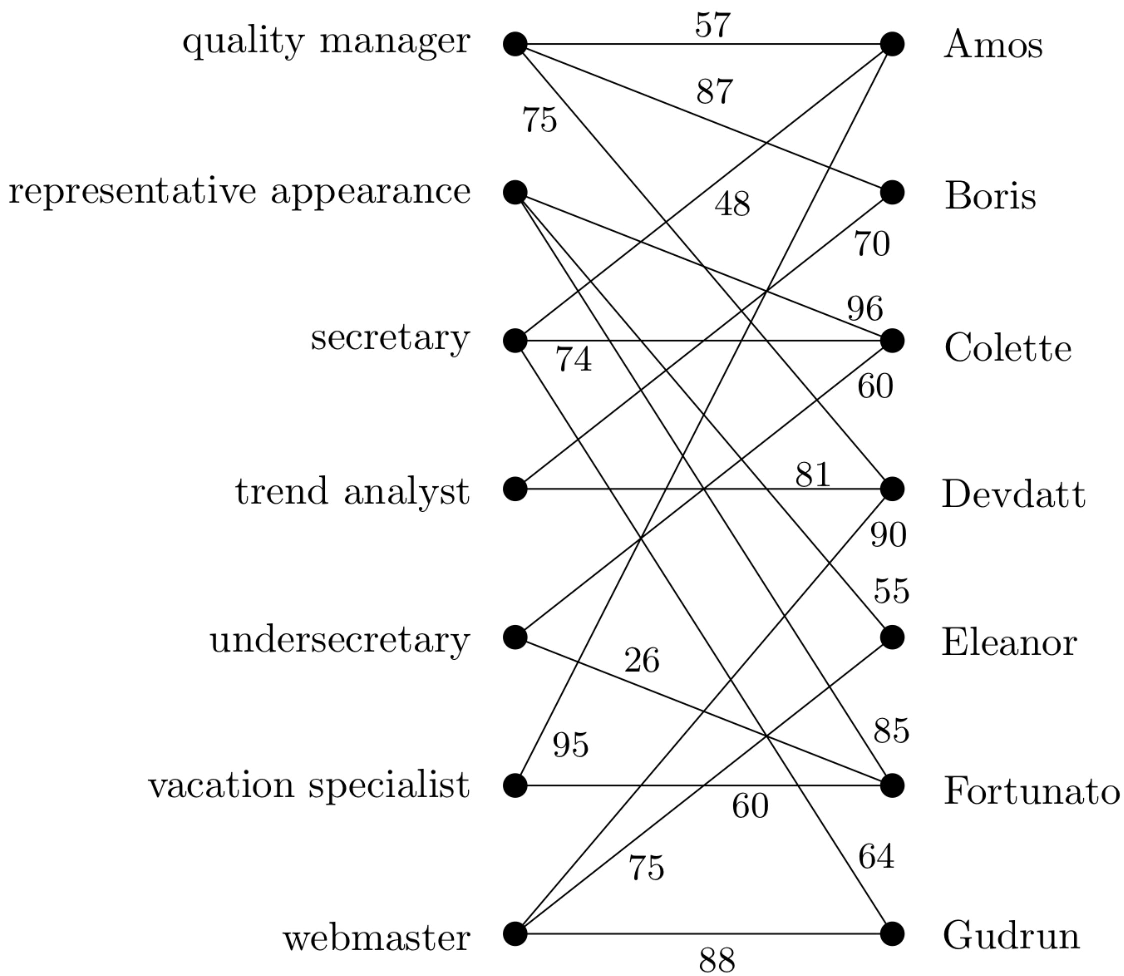
1947: George Dantzig, at the RAND Corporation, creates the **simplex method for linear programming**.

In terms of widespread application, Dantzig's algorithm is one of the most successful of all time: Linear programming dominates the world of industry, where economic survival depends on the ability to optimize within budgetary and other constraints. (Of course, the "real" problems of industry are often nonlinear; the use of linear programming is sometimes dictated by the computational budget.) The simplex method is an elegant way of arriving at optimal answers. Although theoretically susceptible to exponential delays, the algorithm in practice is highly efficient—which in itself says something interesting about the nature of computation.

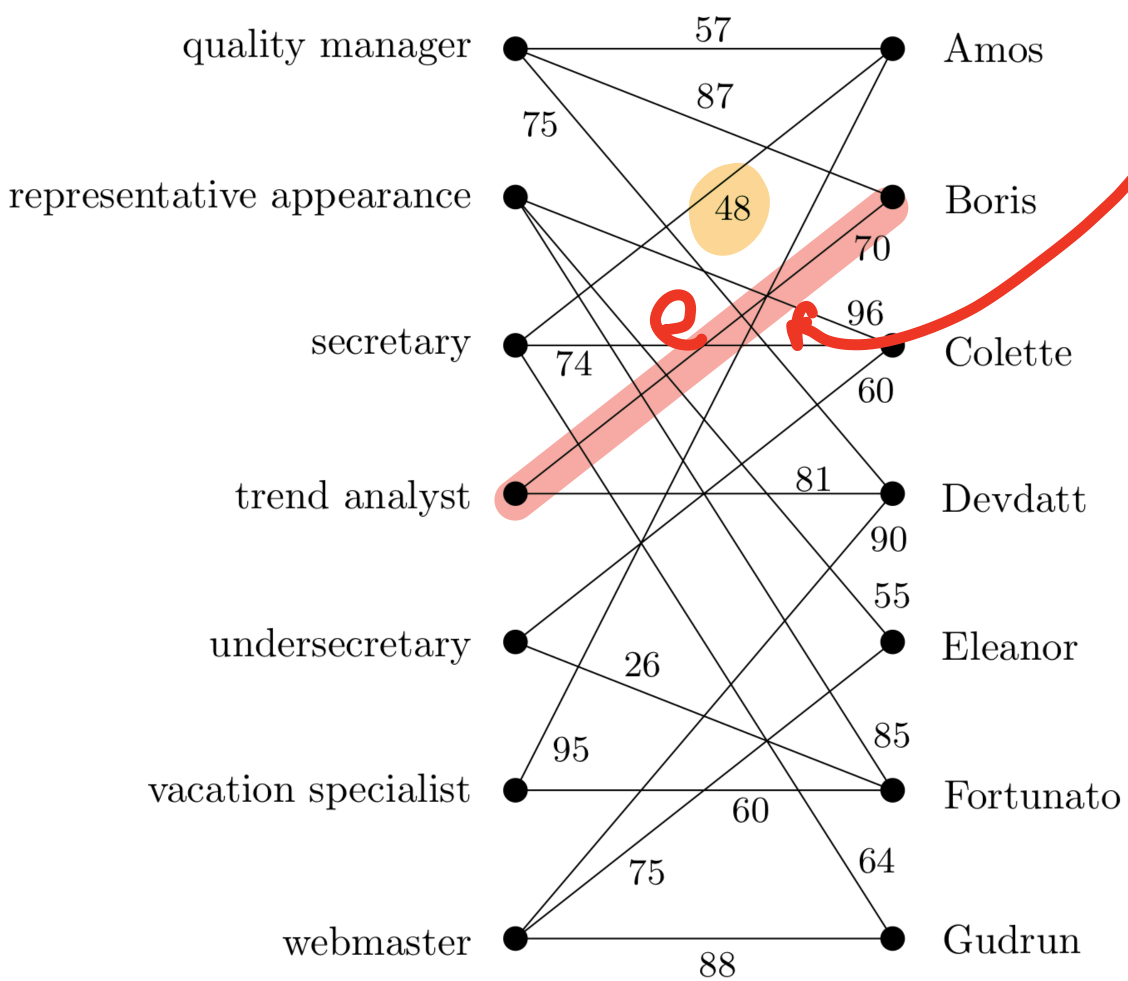
1950: Magnus Hestenes, Eduard Stiefel, and Cornelius Lanczos, all from the Institute for Numerical Analysis at the National Bureau of Standards, initiate the development of **Krylov subspace iteration methods**.

These algorithms address the seemingly simple task of solving equations of the form $Ax = b$. The catch, of course, is that A is a huge $n \times n$ matrix, so that the algebraic answer $x = b/A$ is not so easy to compute. (Indeed, matrix "division" is not a particularly useful concept.) Iterative methods—such as solving equations of

Assignment Problem



Assignment Problem



$x_e = 0$ or 1

$$\max \sum_e w_e x_e$$

s.t. $$\sum_{e,\, v\in e} x_e = 1$$

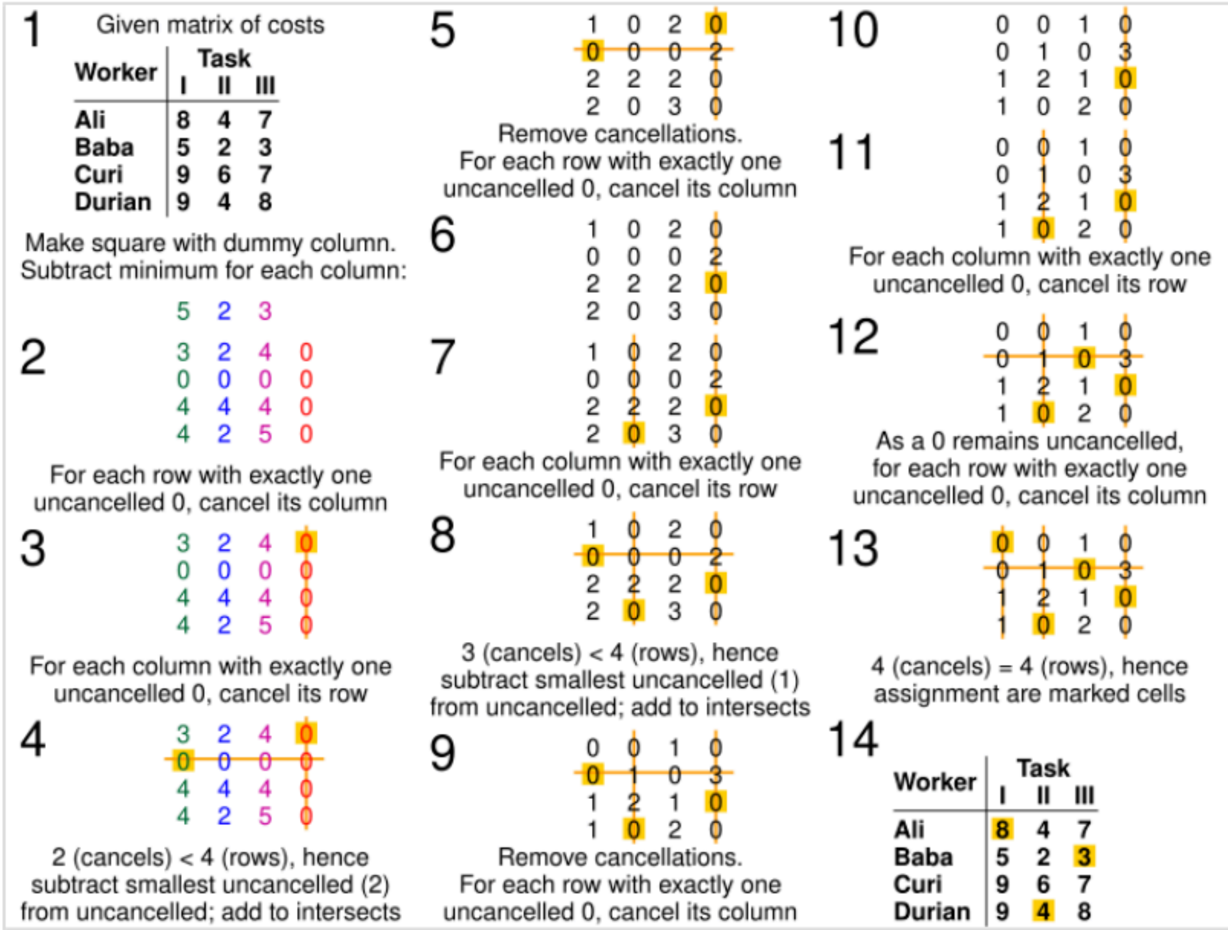
Assignment problem

The **assignment problem** is a fundamental combinatorial optimization problem. In its most general form, the problem is as follows:

> The problem instance has a number of *agents* and a number of *tasks*. Any agent can be assigned to perform any task, incurring some *cost* that may vary depending on the agent-task assignment. It is required to perform as many tasks as possible by assigning at most one agent to each task and at most one task to each agent, in such a way that the *total cost* of the assignment is minimized.

Alternatively, describing the problem using graph theory:

> The assignment problem consists of finding, in a weighted bipartite graph, a matching of maximum size, in which the sum of weights of the edges is minimum.



Worked example of assigning tasks to an unequal number of workers using the Hungarian method

Mathematical Methods of Economics
Author(s): Joel Franklin
Source: *The American Mathematical Monthly*, Vol. 90, No. 4 (Apr., 1983), pp. 229-244

One of these methods is called *linear programming*. I learned about it in 1958. I had just come to Caltech as a junior faculty member associated with the computing center. The director and I made a cross-country trip to survey the most important industrial uses of computers. In New York, we visited the Mobil Oil Company, which had just put in a multi-million-dollar computer system. We found out that Mobil had paid off this huge investment in *two weeks* by doing linear programming.

One surprising thing I found was this: the mathematics was delightful. I knew it was useful, but I hadn't expected it to be beautiful. I was surprised to find that linear programming wasn't just business mathematics or engineering mathematics; it was the general mathematics of linear inequalities. Later I found this mathematics coming into some of my own special fields of research (statistics, numerical analysis, ill-posed problems). Here again, you may have a similar experience.

Undergraduate Texts in Mathematics

Joel Franklin

Methods of Mathematical Economics

Linear and Nonlinear Programming, Fixed-Point Theorems

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for
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