2-minute Research Presentation

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Research interests:

Applied Analysis, modeling
(Calculus of Variations,
Materials Science)

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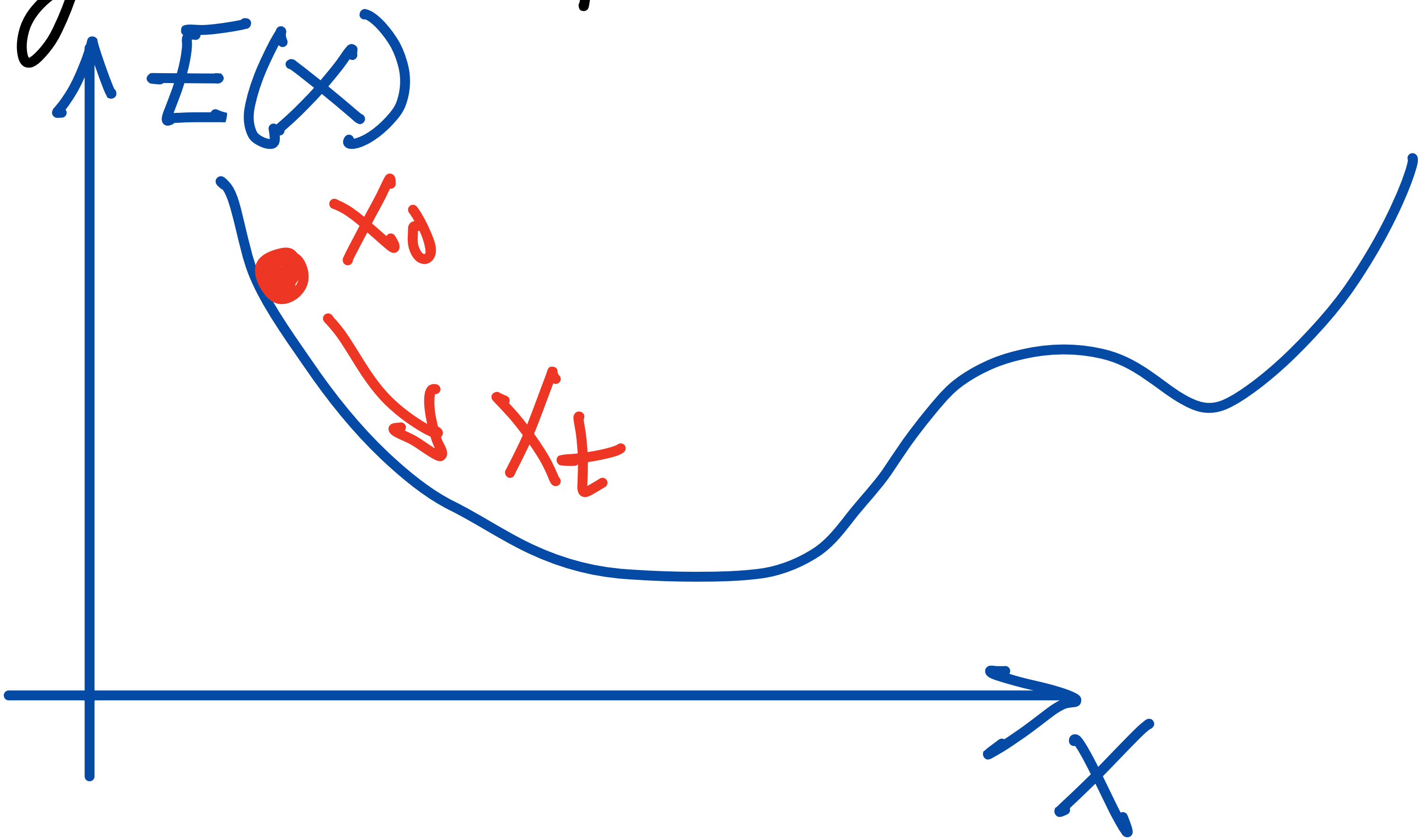
Preferred Interests from Students

Partial Differential Equations

Probability Theory

Differential Geometry

Many Facets of Gradient Flows



$$\boxed{\frac{dX(t)}{dt} = -\nabla E(X(t))}$$

Immediate Consequence of $\dot{x} = -\nabla E(x)$

- $\boxed{E(x(t)) \downarrow \text{ in } t}$

$$\frac{d}{dt} E(x(t)) = \langle \nabla E(x(t)), \dot{x}(t) \rangle$$
$$= -\|\nabla E(x(t))\|^2 \leq 0$$

- $$\boxed{E(t) + \int_0^t \|\nabla E(x(s))\|^2 ds = E(x(0))}$$

Immediate Consequence of $\dot{x} = -\nabla E(x)$

$\boxed{\dot{x} = -\nabla E(x)}$ is in fact equivalent to:

Sandier-Serfaty

$$\boxed{E(x(t)) + \frac{1}{2}\int_0^t \|\nabla E(x(s))\|^2 ds + \frac{1}{2}\int_0^t \|\dot{x}(s)\|^2 ds \leq E(x(0))}$$

Immediate Consequence of $\dot{x} = -\nabla E(x)$

$$\frac{d}{dt} E(x(t))$$

$$= \langle \nabla E(x(t)), \dot{x}(t) \rangle$$

$\dot{x} \parallel -\nabla E(x(t))$

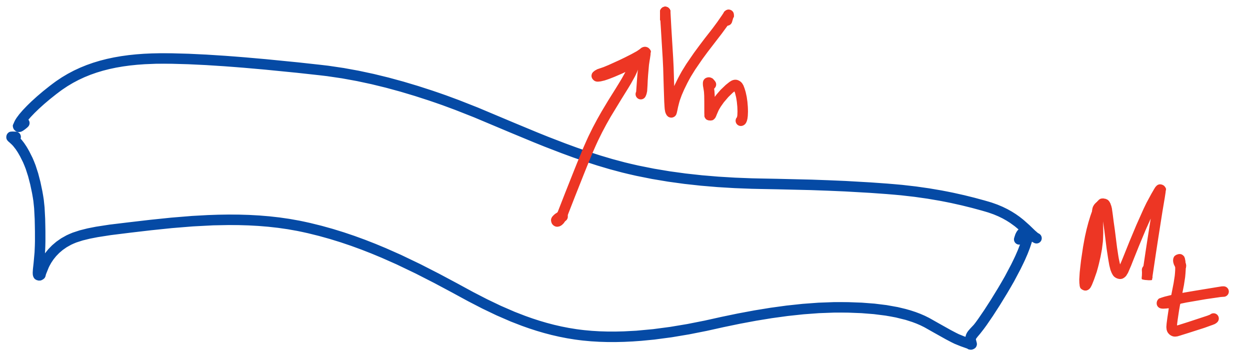
$$\leq -\|\nabla E(x(t))\| \, \|\dot{x}\|$$

$\|\dot{x}\| = \|\nabla E(x)\|^2$

$$= -\frac{1}{2}\|\nabla E(x(t))\|^2 - \frac{1}{2}\|\dot{x}\|^2$$

$\left(ab = -|a||b| = -\frac{1}{2}|a|^2 - \frac{1}{2}|b|^2 \Leftrightarrow a = -b \right)$

Gradient Flows and Geometric Evolutions



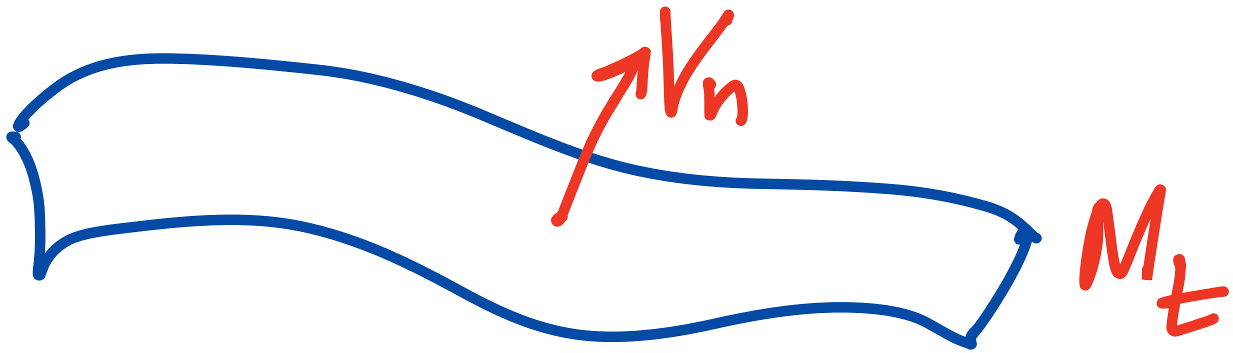
- **Motion by Mean Curvature**

$$V_n = H$$

- **Surface Diffusion**

$$V_n = -\mathrm{div}_S H$$

Gradient Flows and Geometric Evolutions



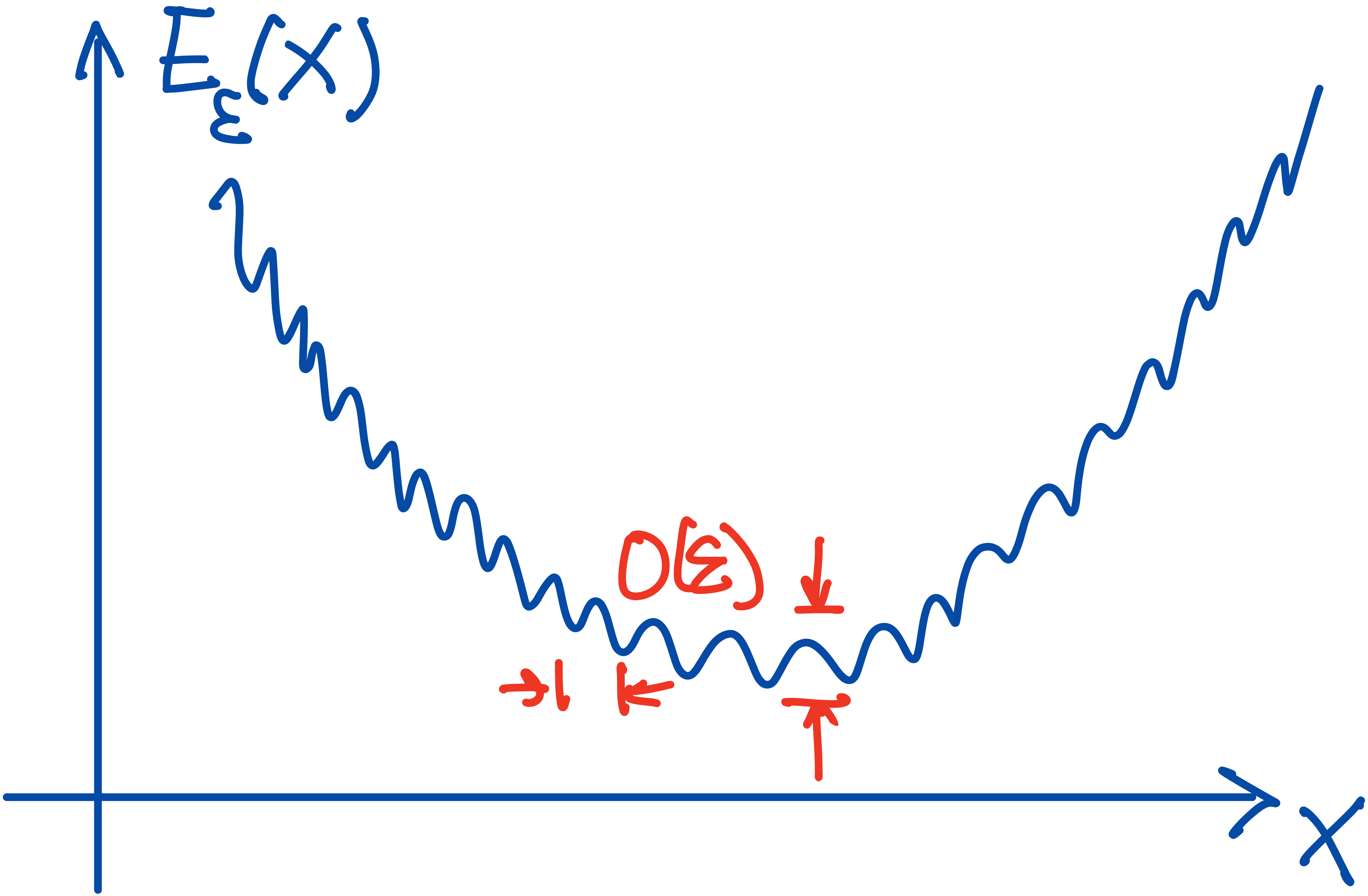
- <u>Motion by Mean Curvature</u>

$$V_n = H \qquad \boxed{V_n = -\nabla_{L^2} E(M(t))}$$

- <u>Surface Diffusion</u>

$$V_n = -\operatorname{div}_S H \qquad \boxed{V_n = -\nabla_{H^{-1}} E(M(t))}$$

Gradient Flow in Inhomogeneous Medium

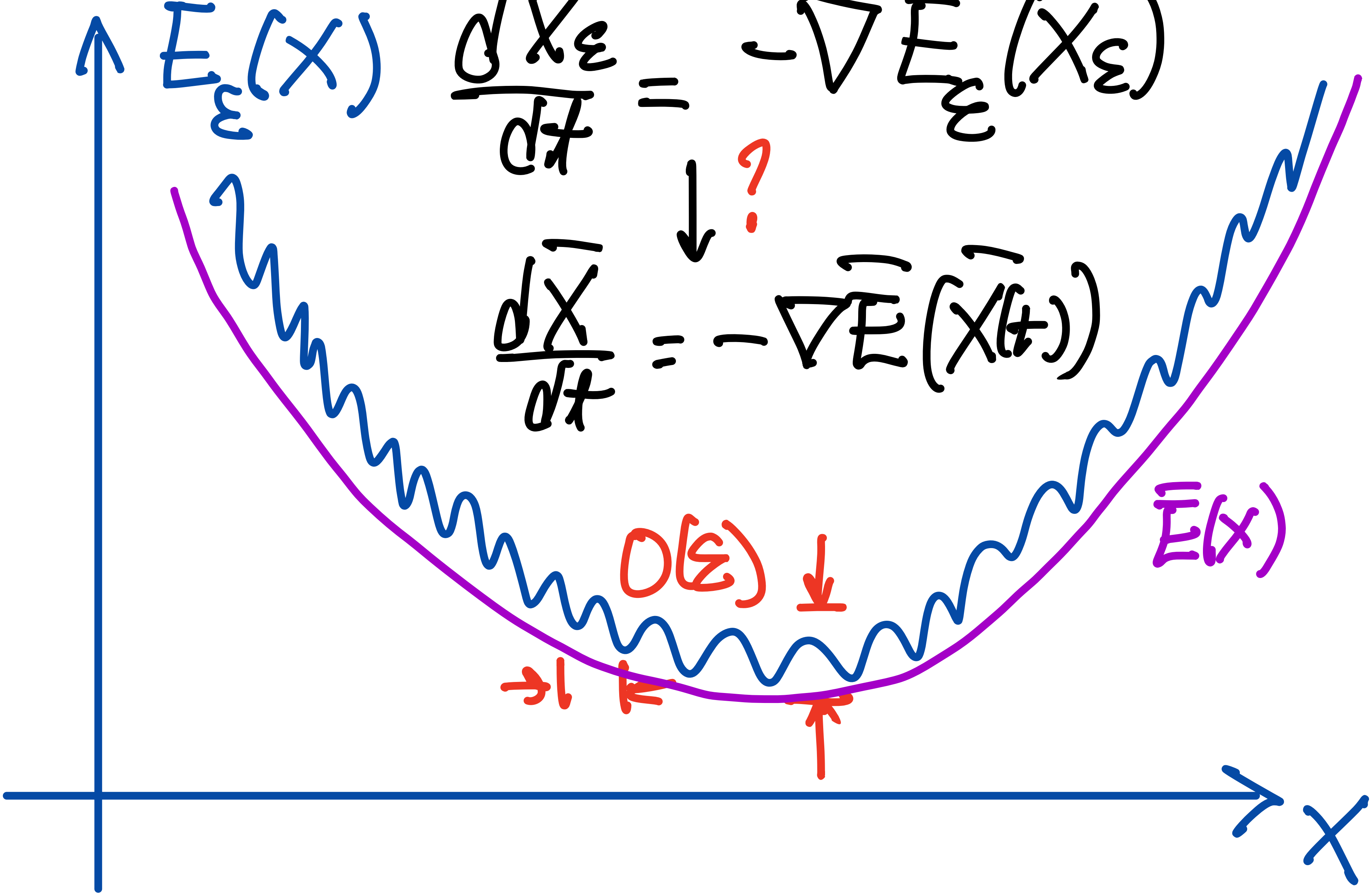


Gradient Flow in Inhomogeneous Medium

$$\frac{dX_\varepsilon}{dt} = -\nabla E_\varepsilon(X_\varepsilon)$$

$$\downarrow ?$$

$$\frac{d\bar{X}}{dt} = -\nabla \bar{E}(\bar{X}(t))$$



Gradient Flow in Inhomogeneous Medium

- Motion of Particle in tilted Periodic Potential (Liang-Y.)

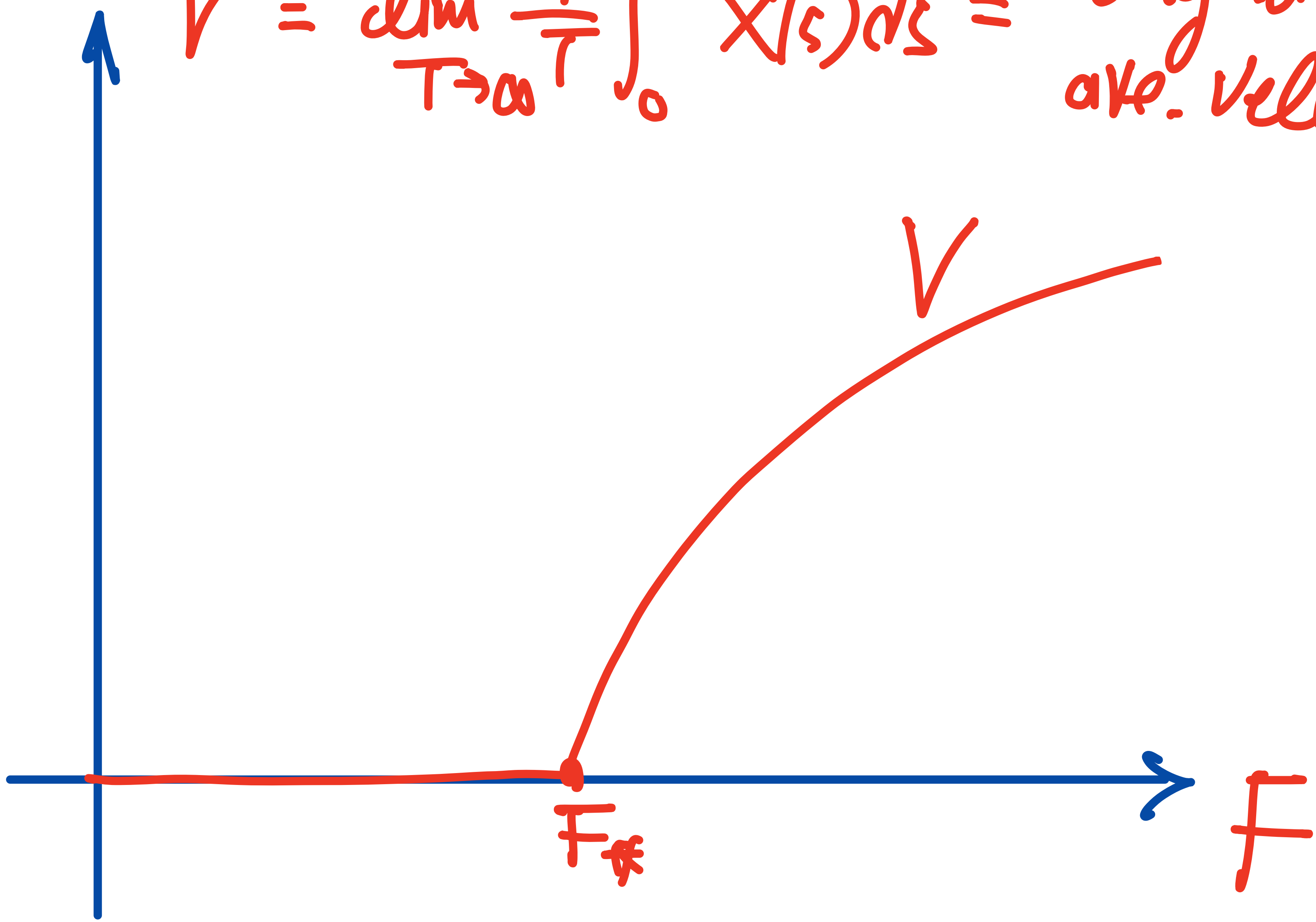
$$\dot{x}(t) = -\sin(x(t)) + F + \dot{W}(t)$$

- Motion by Mean Curvature in Inhomogeneous Medium (Dirr-Y.; Dirr-Karali-Y.)

$$u_t = \Delta u + f(x,u) + F$$

Gradient Flow in Inhomogeneous Medium

$$V = \lim_{T\to\infty} \frac{1}{T}\int_0^T X(s)\,ds = \text{long time ave. velocity}$$



Gradient Flow in Probability Spaces

Heat equation

$$\boxed{\rho_t = \Delta \rho}$$

Gradient Flow in Probability Spaces

Heat equation

$$\boxed{\rho_t = \Delta\rho} \Leftrightarrow \boxed{\rho_t = -\nabla_{L^2} E(\rho)}$$

where

$$\boxed{E(\rho) = \frac{1}{2}\int \|\nabla\rho\|^2 dx \quad \text{(Dirichlet energy)}}$$

Gradient Flow in Probability Spaces

$$\rho_t = \Delta \rho$$

$$= \nabla \cdot (\nabla \rho)$$

$$= \nabla \cdot \left(\rho \frac{\nabla \rho}{\rho}\right)$$

$$= \nabla \cdot (\rho \nabla \log \rho)$$

$$-\nabla_W \mathrm{Ent}(\rho)$$

- $\mathrm{Ent}(\rho) = \int \rho \log \rho \, dx$
- W = Wasserstein Distance (Optimal transport)

Fokker-Planck Equation in Inhomogeneous Medium (Gao-Y.)

SDE

$$\boxed{dX_t = -\nabla U_\varepsilon(X_t)\,dt + \sigma_\varepsilon(X_t) * dW_t}$$

FPE

$$\boxed{\rho_t^\varepsilon = \nabla\cdot\left(B_\varepsilon^{-1}\nabla\rho_t^\varepsilon\right) + \nabla\cdot\left(\rho_t^\varepsilon B_\varepsilon^{-1}\nabla U_\varepsilon\right)}$$

Fokker-Planck Equation in Inhomogeneous Medium (Gao-Y.)

FPE

$$\boxed{\rho_t^\varepsilon = \nabla \cdot \left(B_\varepsilon^{-1} \nabla \rho_t^\varepsilon \right) + \nabla \cdot \left(\rho_t^\varepsilon B_\varepsilon^{-1} \nabla U_\varepsilon \right)}$$

$$= \nabla \cdot \left(\rho_t^\varepsilon B_\varepsilon^{-1} \nabla \left(\log \rho_t^\varepsilon + U_\varepsilon \right) \right)$$

$$\boxed{= -\nabla_{W_\varepsilon} E^\varepsilon(\rho_t^\varepsilon)}$$

Fokker-Planck Equation in Inhomogeneous Medium (Gao-Y.)

FPE

Energy

$$\boxed{E_\varepsilon(\rho) = \int \Big(\underbrace{\rho \log \rho}_{\text{entropy}} + \underbrace{U_\varepsilon(x)\rho}_{\text{potential energy}} \Big) dx}$$

Fokker-Planck Equation in Inhomogeneous Medium (Gao-Y.)

FPE

$$\boxed{\rho_t^\varepsilon = -\nabla_{W_\varepsilon} E^\varepsilon(\rho_t^\varepsilon)}$$

$$\Big\downarrow \varepsilon \to 0$$

$$\boxed{\bar{\rho}_t = -\bar{\nabla}_{\bar{W}}\left(\bar{E}(\bar{\rho}_t)\right)}$$

Stochastic Optimization

Stochastic Gradient Descent

(with Pasupathy et.al.)

$$f(X) = E\,\varphi(X)$$

$$\min_X f(X) \quad \text{vs} \quad \min_X F_n(X)$$

$$F_n(X) = \frac{\varphi(X, w_1) + \varphi(X, w_2) + \cdots + \varphi(X, w_n)}{n}$$

(empirical average)

Stochastic Optimization

Stochastic Gradient Descent

(with Pasupathy et.al.)

$$\boxed{x_{k+1} = x_k - \tau \nabla f(x_k)}$$

vs

$$\boxed{x_{k+1} = x_k - \tau_n \nabla F_n(x_k)}$$

convergence rate?

Thank You
for
Your Attention