- Inverse Matrix and det.

Let $C_{ij} = (-1)^{i+j} \cdot |\tilde{A}_{ij}|$

If $A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$ is invertible, then

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

- Linear System Sol and det (Cramer's Rule)

$$A\vec{x} = \vec{b}$$

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

If $\det(A) \neq 0$, then

$$\vec{x} = A^{-1}\vec{b}$$

$$= \frac{1}{|A|} \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\Rightarrow \quad x_1 = \frac{1}{|A|}\left(C_{11} \cdot b_1 + C_{21} \cdot b_2 + C_{31} \cdot b_3 \right)$$

$$= \frac{1}{|A|}\Big[b_1 \cdot (-1)^{1+1} \cdot |\widetilde{A_{11}}|$$

$$+ b_2 \cdot (-1)^{2+1} \cdot |\widetilde{A_{21}}|$$

$$+ b_3 \cdot (-1)^{3+1} \cdot |\widetilde{A_{31}}| \Big]$$

$$\boxed{A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}}$$

$$x_1 = \frac{1}{|A|} \cdot \begin{vmatrix} b_1 & A_{12} & A_{13} \\ b_2 & A_{22} & A_{23} \\ b_3 & A_{32} & A_{33} \end{vmatrix}$$

$$x_2 = \frac{1}{|A|}\begin{vmatrix} A_{11} & b_1 & A_{13} \\ A_{21} & b_2 & A_{23} \\ A_{31} & b_3 & A_{33} \end{vmatrix}$$

$$x_3 = \frac{1}{|A|}\begin{vmatrix} A_{11} & A_{12} & b_1 \\ A_{21} & A_{22} & b_2 \\ A_{31} & A_{32} & b_3 \end{vmatrix} \qquad ax = b \qquad x = \frac{b}{a}$$

- Change of coordinates in integration

① Polar Coordinates $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

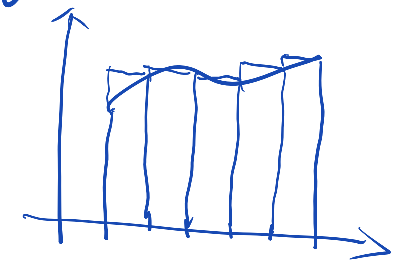
$$\iint_{x^2+y^2\le 1} f(x,y)\,dx\,dy = \int_0^{2\pi}\int_0^1 f(r,\theta)\, r\,dr\,d\theta$$

1) Jacobian Matrix is the first order derivative of a vector-valued multivariable function.

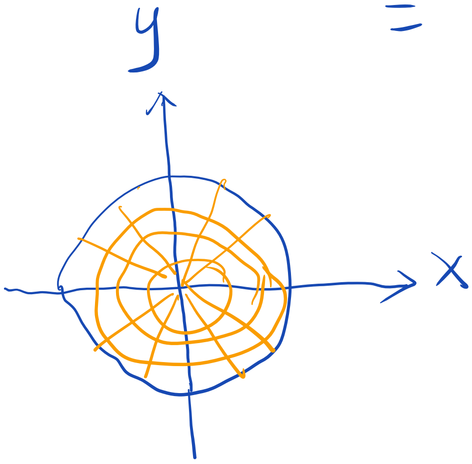
$$\vec{X}(r,\theta) = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix}$$

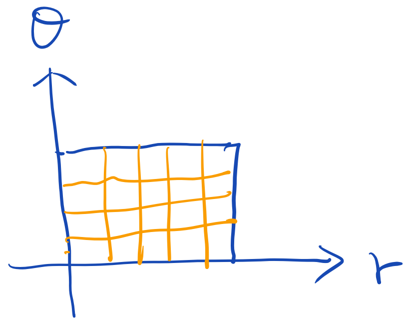
$$J = \begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix}$$

$$|J| = r\cos^2\theta + r\sin^2\theta = r$$



$$dx\,dy = |J|\,dr\,d\theta$$
$$= r\,dr\,d\theta$$





$$\approx |J|\,\Delta\theta\,\Delta r$$

$$\left(dx = \frac{\partial x}{\partial r}dr + \frac{\partial x}{\partial \theta}d\theta \right)$$

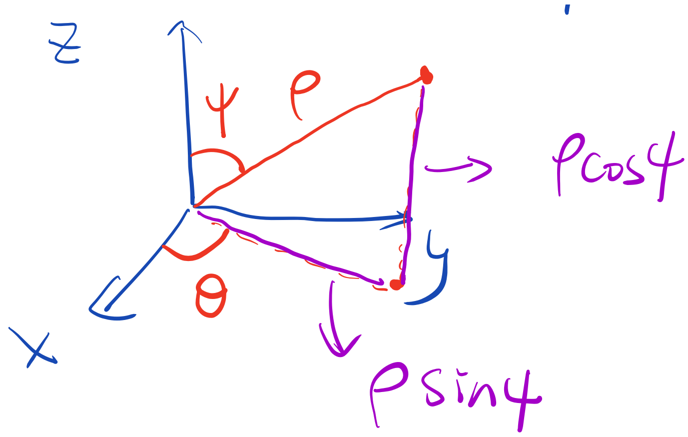
$$x = r\cos\theta \Rightarrow \Delta x \approx \frac{\partial x}{\partial r}\Delta r + \frac{\partial x}{\partial \theta}\Delta\theta$$

$$x = r\sin\theta \Rightarrow \Delta y \approx \frac{\partial y}{\partial r}\Delta r + \frac{\partial y}{\partial \theta}\Delta\theta$$

$$\left(dy = \frac{\partial y}{\partial r}dr + \frac{\partial y}{\partial \theta}d\theta \right)$$

② Spherical Coordinate

$$\begin{cases} x = \rho\sin\phi\cos\theta \\ y = \rho\sin\phi\sin\theta \\ z = \rho\cos\phi \end{cases}$$



$$dx\,dy\,dz = |J|\,d\rho\,d\psi\,d\theta$$

$$|J| = \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial x}{\partial \psi} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \psi} & \frac{\partial y}{\partial \theta} \\ \frac{\partial z}{\partial \rho} & \frac{\partial z}{\partial \psi} & \frac{\partial z}{\partial \theta} \end{vmatrix}$$