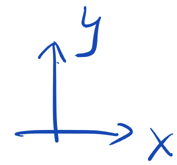
Vectors

- $\mathbb{R}$ is the set of all real numbers

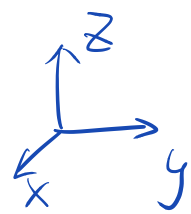
  $\underbrace{\forall}_{\text{for any}}\ a \underbrace{\in}_{\text{in}} \mathbb{R}$: for any real number a

- $\mathbb{C}$ is the set of all complex members.

- $\mathbb{R}^2 = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : \forall x, y \in \mathbb{R} \right\}$



- $\mathbb{R}^3 = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : \forall x, y, z \in \mathbb{R} \right\}$



- $\mathbb{R}^4 = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} : \forall x, y, z, w \in \mathbb{R} \right\}$

- $\mathbb{R}^{100} = \left\{ \begin{pmatrix} x_1 \\ \vdots \\ x_{100} \end{pmatrix} : \forall x_i \in \mathbb{R}, i \right\}$

- Scalars mean numbers in $\mathbb{R}$ (or $\mathbb{C}$)

- Scalar Multiplication $c \in \mathbb{R}$, $\vec{v} \in \mathbb{R}^n$ $(n = 2, 3, 4, \ldots)$

  $\vec{v} = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$, $c\vec{v} = \begin{pmatrix} cv_1 \\ \vdots \\ cv_n \end{pmatrix}$

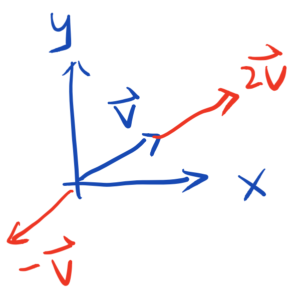
- Vector Addition : $\vec{v}, \vec{w} \in \mathbb{R}^n$

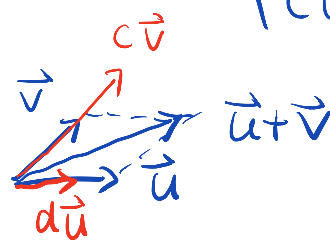
$$\vec{v} + \vec{w} = \begin{pmatrix} v_1 + w_1 \\ \vdots \\ v_n + w_n \end{pmatrix}$$

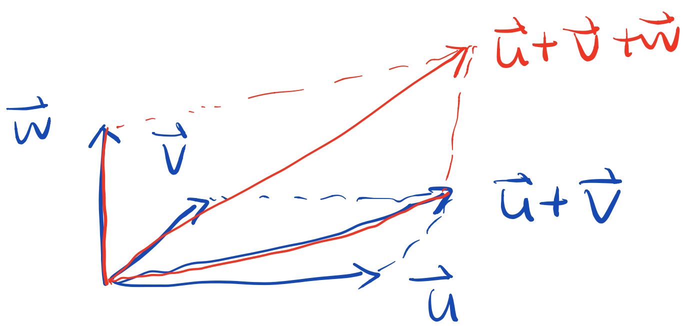
- Linear Combination

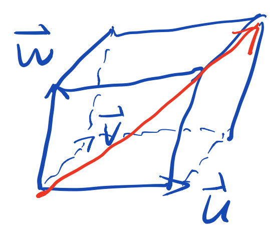
$$c\vec{v} + d\vec{w} = \begin{pmatrix} cv_1 + dw_1 \\ \vdots \\ cv_n + dw_n \end{pmatrix}$$

$$c\vec{u} + d\vec{v} + e\vec{w} = \begin{pmatrix} cu_1 + dv_1 + ew_1 \\ \vdots \\ cu_n + dv_n + ew_n \end{pmatrix}$$









- Dot Product $\quad \vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \quad \vec{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$

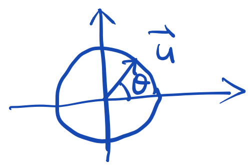
$$\vec{v} \cdot \vec{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$$

- Length $\quad \|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}} = \sqrt{v_1^2 + v_2^2 + v_3^2}$

$$\left\|\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}\right\| = \sqrt{v_1^2+v_2^2}$$

- Unit vector: if $\|\vec{v}\|=1$

in $\mathbb{R}^2$: $\vec{i}=\begin{pmatrix}1\\0\end{pmatrix}$, $\vec{j}=\begin{pmatrix}0\\1\end{pmatrix}$, $\vec{u}=\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}$



in $\mathbb{R}^3$: $\vec{i}=\begin{pmatrix}1\\0\\0\end{pmatrix}$, $\vec{j}=\begin{pmatrix}0\\1\\0\end{pmatrix}$, $\vec{k}=\begin{pmatrix}0\\0\\1\end{pmatrix}$

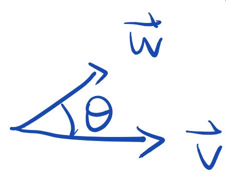
$\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^3$

What is the pic of all linear comb.

of $\begin{cases} c\vec{u} \\ c\vec{u}+d\vec{v} \\ c\vec{u}+d\vec{v}+e\vec{w} \end{cases}$

- $\dfrac{\vec{v}}{\|\vec{v}\|} = \dfrac{1}{\|\vec{v}\|}\cdot\vec{v}$ is alway unit, parallel to $\vec{v}$.

- Formula $\vec{v}\cdot\vec{w} = \|\vec{v}\|\cdot\|\vec{w}\|\cdot\cos\theta$

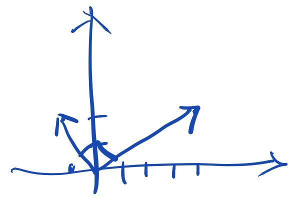


- Assume $\vec{v}\neq\vec{0}$, $\vec{w}\neq\vec{0}$.

$\vec{v}\cdot\vec{w}=0 \iff \cos\theta=0 \iff \theta=90^\circ$

We say $\vec{v}, \vec{w}$ are perpendicular.

$\begin{pmatrix}4\\2\end{pmatrix}\cdot\begin{pmatrix}-1\\2\end{pmatrix}=0$

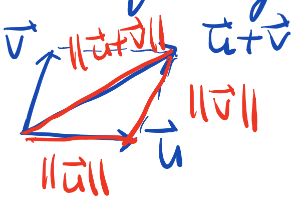


- Schartz Inequality

$$|\vec{v}\cdot\vec{w}| = \|\vec{v}\|\cdot\|\vec{w}\|\cdot|\cos\theta| \le \|\vec{v}\|\cdot\|\vec{w}\|$$

$$|v_1w_1+v_2w_2+v_3w_3| \le \sqrt{v_1^2+v_2^2+v_3^2}\sqrt{w_1^2+w_2^2+w_3^2}$$

- Triangle Inequality $\|\vec{u}+\vec{v}\| \le \|\vec{u}\|+\|\vec{v}\|$



$$\sqrt{(u_1+v_1)^2+(u_2+v_2)^2+(u_3+v_3)^2} \le \sqrt{u_1^2+u_2^2+u_3^2}+\sqrt{v_1^2+v_2^2+v_3^2}$$

- Matrix $\mathbb{R}^{m\times n}$

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m\times n} = (a_{\underline{ij}})_{m\times n}$$

i-th row
j-th col

$$B=\begin{pmatrix}1&2&3\\4&5&6\\7&8&9\end{pmatrix} \qquad B_{32}=8$$

- $$\underbrace{x\begin{pmatrix}1\\-1\\0\end{pmatrix} + y\begin{pmatrix}0\\1\\-1\end{pmatrix} + z\begin{pmatrix}0\\0\\1\end{pmatrix}} = \begin{pmatrix}x\\y-x\\z-y\end{pmatrix}$$

$\Downarrow$ we define it as Matrix-Vector Multiplication

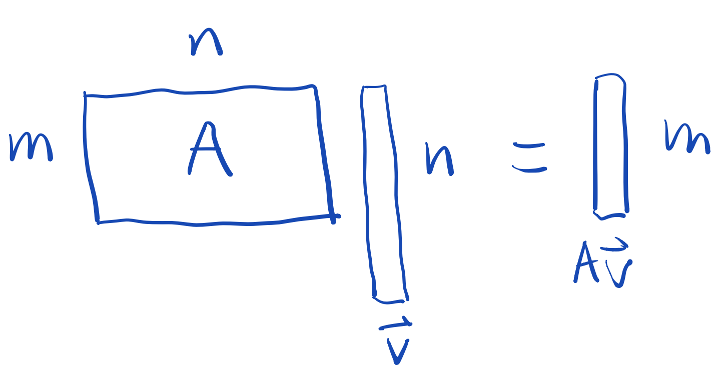
$$\begin{pmatrix}1 & 0 & 0\\-1 & 1 & 0\\0 & -1 & 1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}$$

It is also equivalent to doing dot products of rows in the matrix with $\begin{pmatrix}x\\y\\z\end{pmatrix}$

$$\begin{pmatrix}1 & 0 & 0\\-1 & 1 & 0\\0 & -1 & 1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}(1\ 0\ 0)\cdot\begin{pmatrix}x\\y\\z\end{pmatrix}\\(-1\ 1\ 0)\cdot\begin{pmatrix}x\\y\\z\end{pmatrix}\\(0\ -1\ 0)\cdot\begin{pmatrix}x\\y\\z\end{pmatrix}\end{pmatrix} = \begin{pmatrix}x\\-x+y\\-y+z\end{pmatrix}$$

- $A \in \mathbb{R}^{m\times n}$, $\vec{v} \in \mathbb{R}^n$

$A\vec{v} \in \mathbb{R}^m$



$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = \begin{pmatrix} v_1 + 2v_2 + 3v_3 + 4v_4 \\ 5v_1 + 6v_2 + 7v_3 + 8v_4 \end{pmatrix}$$