MA 351 Section 041/011
Fall 2023

Homework 2

Due on **Sep 6 before 9am** on gradescope.

To receive full credit, show necessary reasoning unless it's straightforward computation.

1. (10 pts) Find all solutions to the equation $x + y - z - w = 0$.

2. (10 pts) Find the point-direction form of the line equation for the intersection line of two planes described by $-2x + 3y + z = 0$ and $x + z = 3$

3. (20 pts) Find the intersection of two hyperplanes (3-dimensional plane in a 4D space) described by $-2x+3y+z-w = 2$ and $-2x+4y+3z+w = 3$. It suffices to simply write out all solutions to the linear system.

4. (20 pts) To find the intersection of the three planes described by equations $2y + 4z = 2$, $-2x + 3y + z = 2$, and $-4x + 4y + 2z = 3$, we can set it up as a linear system $A\vec{x} = \vec{b}$. Find the matrix A and its inverse A^{-1} by Gaussian Elimination. Verify your computation by computing both AA^{-1} and $A^{-1}A$. Finally use A^{-1} to find the solution to the linear system $A\vec{x} = \vec{b}$.

5. (20 pts) Determine whether the following matrix is invertible. If yes, find its inverse and verify your computation by computing AA^{-1}.

$$A = \begin{pmatrix} 0 & 2 & 4 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 4 & 3 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

6. (20 pts)

Definition 1. *Upper triangular matrices.* A square matrix $A \in \mathbb{R}^{n\times n}$ is called *upper triangular* if all entries below the diagonal (A_{ii} entries) are all zeros. For example, $A = \begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix}$ is an upper triangular matrix, where a, b, c, d, e, f can be any numbers (whether they are zeros or not).

Fact: if an upper triangular matrix A is invertible, then A^{-1} is also upper triangular. Convince yourself this fact by computing the inverse of the following matrix:

$$A = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$