Homework 7

Due on **Oct 25 before 9am** on gradescope.

To receive full credit, show necessary reasoning unless it's straightforward computation.

1. (10 pts) Use the cofactor matrix and determinant to find the $(2,3)$-entry of A^{-1}, where
$$A = \begin{pmatrix} 1 & 0 & -2 & 1 \\ -1 & 0 & 1 & -2 \\ -1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 2 \end{pmatrix}.$$

2. (20 pts) The spherical coordinates are given as
$$\begin{cases} x = \rho \sin\phi \cos\theta \\ y = \rho \sin\phi \sin\theta \\ z = \rho \cos\phi \end{cases}.$$
The Jacobian matrix of the change of coordinates of using spherical coordinates is the 3×3 matrix of first order derivatives
$$J = \begin{pmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial x}{\partial \phi} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \phi} & \frac{\partial y}{\partial \theta} \\ \frac{\partial z}{\partial \rho} & \frac{\partial z}{\partial \phi} & \frac{\partial z}{\partial \theta} \end{pmatrix}.$$
Find the determinant of the Jacobian matrix.

3. (10 pts) The cylindrical coordinates are (r, θ, z) given as
$$\begin{cases} x = r \cos\theta \\ y = r \sin\theta \\ z = z \end{cases}.$$
The Jacobian matrix of the change of coordinates of using cylindrical coordinates is the 3×3 matrix of first order derivatives
$$J = \begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial z} \end{pmatrix}.$$

Find the determinant of the Jacobian matrix.

4. (20 pts) Use Cramer's Rule to find z satisfying

$$\begin{pmatrix} 1 & 0 & -2 & 1 \\ -1 & 0 & 1 & -2 \\ -1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}.$$

5. (20 pts) For the matrix $A = \begin{pmatrix} 1 & 0 & 2 \\ -1 & 2 & 2 \\ 0 & 0 & -1 \end{pmatrix}$,

- Find all its eigenvalues.
- For each eigenvalue, find one eigenvector.
- For each eigenvalue, find its eigenspace.

6. (20 pts) For the matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$, find all its eigenvalues. For each eigenvalue, find the eigenspace.