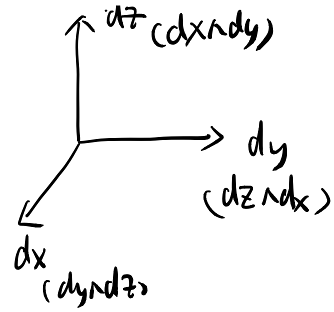
0-form $\quad f(x,y,z)$

1-form $\quad f\,dx + g\,dy + h\,dz \sim \langle f, g, h\rangle$

2-form $\quad f\,dy\wedge dz + g\,dz\wedge dx + h\,dx\wedge dy \sim \underline{\underline{\langle f, g, h\rangle}}$

3-form $\quad f\,dx\wedge dy\wedge dz$



Assume coefs are C^1 on the region R:

Then a n-form α is $\begin{cases} \text{closed} & \text{if } d\alpha = 0 \\ \text{exact} & \text{if } \alpha = d\beta \text{ for } (n-1)\text{-form } \beta \end{cases}$ $\quad d(f\alpha) = df\wedge d\alpha + f\,d\alpha$

$d(F\,dx + G\,dy + H\,dz)$

Theorems : ① exact $\Rightarrow$ closed $\quad \nabla\times\langle F,G,H\rangle \quad \nabla\cdot\langle F,G,H\rangle$

$\alpha = d\beta \Rightarrow d\alpha = d(d\beta) = 0 \quad d(F\,dy\wedge dz + G\,dz\wedge dx + H\,dx\wedge dy)$

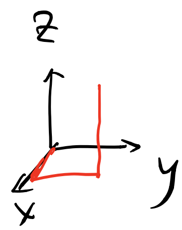
② $d^2 = 0 \quad (\underline{\nabla\times\nabla f = \vec{0}}\ ;\ \underline{\nabla\cdot\nabla\times\langle F,G,H\rangle} = 0)$

③ closed $\Rightarrow$ exact (Poincare's Lemma)

1-form $\quad F\,dx + G\,dy + H\,dz = df$

$$\langle F,G,H\rangle = \nabla f$$

$$\nabla\times\langle F,G,H\rangle = 0 \Rightarrow f \text{ exists}$$



$$f(x,y,z) = \int_0^x F(t,0,0)\,dt + \int_0^y G(x,t,0)\,dt + \int_0^z H(x,y,t)\,dt$$

$$\text{Scalar function} \xrightarrow[\text{grad}]{\nabla f} \text{Vector field} \xrightarrow{\nabla\times} \text{Vector field} \xrightarrow{\nabla\cdot} \text{scalar function}$$

$\langle f_x, f_y, f_z\rangle$

$$\text{0-form} \xrightarrow{d} \text{1-form} \xrightarrow{d} \text{2-form} \xrightarrow{d} \text{3-form}$$

$$df = f_x\,dx + f_y\,dy + f_z\,dz$$

$$d(F\,dx + G\,dy + H\,dz) = \underline{[H_y - G_z]\,dy\wedge dz + [F_z - H_x]\,dz\wedge dx + [G_x - F_y]\,dx\wedge dy}$$

$$\nabla\times\langle F,G,H\rangle = \begin{vmatrix} i & j & k \\ \partial x & \partial y & \partial z \\ F & G & H \end{vmatrix} = i\begin{vmatrix} \partial y & \partial z \\ G & H \end{vmatrix} - j\begin{vmatrix} \partial x & \partial z \\ F & H \end{vmatrix}$$

$+ k \,|\, \partial x \quad \partial y \,|$

$+ k \begin{vmatrix} F & G \end{vmatrix}$

$$d[F\,dy\wedge dz + G\,dz\wedge dx + H\,dx\wedge dy] = (F_x + G_y + H_z)\,dx\wedge dy\wedge dz$$

Midterm Practice Problems (on Blackboard)

2. $d[F(x,y)\,dx + G(x,y)\,dy] = 0 \Leftrightarrow \nabla\times\langle F, G, 0\rangle = \vec{0}$

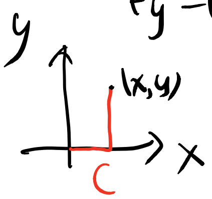
$$\Updownarrow$$

$$\langle 0, 0, G_x - F_y\rangle = \vec{0}$$

$$\Updownarrow$$

$$F_y = G_x$$

$F_y = G_x \Rightarrow F\,dx + G\,dy = df$



$\int_C F\,dx + G\,dy$

$$f(x,y) = \int_0^x F(t,0)\,dt + \int_0^y G(x,t)\,dt$$

(a) $\underbrace{(3+2xy)}_{F}dx + \underbrace{(x^2-3y^2)}_{G}dy$

$F_y = 2x \qquad G_x = 2x \quad \Rightarrow F_y = G_x \Rightarrow$ closed

$\Rightarrow$ exact

$\Rightarrow f$ exists

$$f(x,y) = \int_0^x F(t,0)\,dt + \int_0^y G(x,t)\,dt$$

$$= \int_0^x (3 + 2\cdot t\cdot 0)\,dt + \int_0^y (x^2 - 3t^2)\,dt$$

$$= 3x + x^2y - t^3\Big|_0^y = 3x + x^2y - y^3$$

$$\boxed{\nabla f = \langle f_x, f_y\rangle = \langle 3+2xy,\ x^2-3y^2\rangle} \Rightarrow df$$

$= (3+2xy)dx$

$+ (x^2-3y^2)dy$

(b) $\underbrace{(ye^x + \sin y)}_{F}dx + \underbrace{(e^x + x\cos y)}_{G}dy$

$F_y = e^x + \cos y \qquad G_x = e^x + \cos y$

3. (a) $(3+2xy)dx + (x^2-3y^2)dy + 1\cdot dz$

$F\,dx \quad + \quad G\,dy \quad + \quad H\,dz$

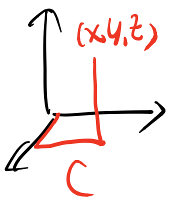
$$d(F\,dx + G\,dy + H\,dz) = [H_y - G_z]\,dy\wedge dz + [F_z - H_x]\,dz\wedge dx + [G_x - F_y]\,dx\wedge dy$$

$$= 0\cdot dy\wedge dz + 0\cdot dz\wedge dx + [2x - 2x]\,dx\wedge dy$$

$$= 0$$

$\Rightarrow$ 1-form is closed $\Rightarrow$ exact $\Rightarrow f$ exists

$\int_C F\,dx + G\,dy + H\,dz$



$$f(x,y,z) = \int_0^x F(t,0,0)\,dt + \int_0^y G(x,t,0)\,dt + \int_0^z H(x,y,t)\,dt$$

$$= \int_0^x (3 + 2\cdot t\cdot 0)\,dt + \int_0^y (x^2 - 3t^2)\,dt + \int_0^z 1\cdot dt$$

$$= 3x \quad + x^2 y - t^3\Big|_0^y + z$$

$$= 3x \quad + x^2 y - y^3 + z$$

$$\boxed{\nabla f = \langle f_x, f_y, f_z\rangle = \langle \underline{3 \quad + 2xy}, x^2 - 3y^2, 1\rangle}$$

(b)