Homework 1

Due on Jan 22nd in class.

1. Find all vectors in $\mathbb{R}^3$ perpendicular to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{i} - \mathbf{j} + \mathbf{k}$.

2. Find the projection of the vector $(1, 2, 3)$ along the vector $(1, 1, 1)$.

3. Find the volume of the parallelepiped spanned by $(1, 1, 1)$, $(1, -2, 3)$, and $(-3, 2, -1)$.

4. Consider the intersection line of the two planes with equations $x + 2y + 4z = 7$ and $4x + 2y + z = 7$. Write down the line equation in the point-direction form.

5. Sketch the region given in spherical coordinates by the inequalities

$$0 \le \rho \le 1, \ 0 \le \theta \le \pi/2, \ 0 \le \phi \le \pi.$$

Express this region in cylindrical coordinates.

6. Expand and simplify $\|\mathbf{u} + \mathbf{v}\|^2 - \|\mathbf{u} - \mathbf{v}\|^2$.

7. Find the projection of the point $(2, 3, 5)$ on the plane $x + 2y + 4z = 7$.