Homework 4

Due on Feb 12th in class.

1. (20 points) Compute the second-order Taylor formula of the function

\[ f(x, y) = e^{x+y^2} \cos(x - y) + \sin(xy) + \sin(x^3 + y^3). \]

around the point \((0, 0)\).

2. (60 points) For each of the following functions, find all the critical points, and decide if each one is a local maximum, a local minimum, or neither.

(a) \( f(x, y) = x^2 + xy - x + y^2 + 4y + 7 \).

(b) \( f(x, y) = x^4 + y^{10} \).

(c) \( f(x, y) = -x^6 - y^8 \).

(d) \( f(x, y) = x^4 + y^3 \).

(e) \( f(x, y) = x^7y^6 \).

3. (20 points) Consider the equation

\[ \sin(xyz) + x + y^2 + z^3 = 0. \]

Simply apply

(a) Is Implicit Function Theorem satisfied to assert that there a differentiable function \( f \) such that \( x = f(y, z) \) solves the equation near \((0, 0, 0)\)? If so, find \( \partial_y f(0, 0) \) and \( \partial_z f(0, 0) \).

(b) Is Implicit Function Theorem satisfied to assert that there a differentiable function \( g \) such that \( y = f(x, z) \) solves the equation near \((0, 0, 0)\)? If so, find \( \partial_x g(0, 0) \) and \( \partial_z g(0, 0) \).