Homework 5

Due on Feb 26 in class.

1. Find the perimeter of an ellipse $\frac{1}{4}x^2 + \frac{1}{9}y^2 = 1$ by computing the arc length of the following parametric curve $\mathbf{c}(t)$. Only write down the integral and simplify it as much as possible. **Do not evaluate the integral by trying to find its antiderivative**.

$$x(t) = 2\cos t, \quad y = 3\sin t, \quad 0 \le t \le 2\pi.$$

2. Let $\mathbf{F} = (x^2, x^2y, z + zx)$. Find $curl\,\mathbf{F}$. Verify that $\nabla \cdot (\nabla \times \mathbf{F}) = 0$.

3. Evaluate the integral $\int_C \omega$ with differential form $\omega = \frac{-y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy$ and the following two curves C:

   (a) $x = \cos t, y = \sin t, t \in [0, 2\pi]$.

   (b) $x = \cos(-t), y = \sin(-t), -2\pi \le t \le 0$.

4. Determine which of the following 1-forms on $\mathbb{R}^2$ are exact. Express the exact 1-forms in the form df. In other words, find $f(x, y)$ for exact forms.

   (a) $3ydx + xdy$

   (b) $ydx + xdy$

   (c) $e^x ydx + e^x dy$

   (d) $-ydx + xdy$

5. Calculate $\int_{C_1} zdx + xdz$ and $\int_{C_2} zdx + xdz$ where the two curves are given as:

   (a) C_1 is the helix $x = \cos t, y = \sin t, z = t, 0 \le t \le 2\pi$.

   (b) C_2 is the line segment connecting $(0, 0, 0)$ and $(0, 0, 2\pi)$. (Hint: find the line equation first then use it as the parametric curve for C_2).