Homework 6

Due on Mar 4 in class.

1. (20 pts) It is sometimes possible to make a 1-form exact by multiplying
it by a nonzero function of $x$ and $y$ called an integrating factor. For
the following 1-forms, in each of the nonexact ones, find an integrating
factor of the form $x^n$ for some integer $n$.

(a) $3ydx + xdy$

(b) $\cos x \cos ydx - \sin x \sin ydy$

(c) $-ydx + xdy$

2. (20 pts)

(a) An immediate consequence of Green’s theorem is that the area of
a rectangle enclosed by $C$ (with counterclockwise orientation) is
$\int_C xdy$. Check this by direct calculation for a rectangle with ver-
tices $(0, 0), (a, 0), (a, b), (0, b)$.

(b) Let $C$ be a circle of radius $r$ centered at $(0, 0)$ oriented counterclock-
wise. Check that $\int_C xdy$ gives the area inside the circle.

3. (40 pts) Let $r, \theta$ be polar coordinates, so $x = r\cos \theta, y = r\sin \theta$.

(a) (5 pts) Convert $dx$ and $dy$ to polar coordinates.

(b) (30 pts) Use (a) to calculate $\int_C dx + dy$ where $C$ is given by
$r = \sin \theta, \ 0 \leq \theta \leq \pi$.

(c) (5 pts) Use the result in (a) to solve for $dr$ and $d\theta$.

4. (20 pts) Let $f$ be a function such that

$$\nabla f = \left( \frac{1}{2} - \frac{y^2}{2x^2}, \frac{y}{x} \right)$$

for $x > 0$. Find the function $f(x, y)$ by computing some line integral
along a piece-wise line segment path from \((a,0)\) to \((x,y)\) with \(a > 0\) being some fixed constant. **No credit at all if you only give some function.**