Homework 6

Due on Mar 4 in class.

1. (20 pts) It is sometimes possible to make a 1-form exact by multiplying it by a nonzero function of x and y called an integrating factor. For the following 1-forms, in each of the nonexact ones, find an integrating factor of the form x^n for some integer n.
   (a) $3ydx + xdy$

   (b) $\cos x \cos ydx - \sin x \sin ydy$

   (c) $-ydx + xdy$

2. (20 pts)
   (a) An immediate consequence of Green's theorem is that the area of a rectangle enclosed by C (with counterclockwise orientation) is $\int_C xdy$. Check this by direct calculation for a rectangle with vertices $(0, 0), (a, 0), (a, b), (0, b)$.

   (b) Let C be a circle of radius r centered at $(0, 0)$ oriented counterclockwise. Check that $\int_C xdy$ gives the area inside the circle.

3. (40 pts) Let r, θ be polar coordinates, so $x = rcos\theta$, $y = rsin\theta$.
   (a) (5 pts) Convert dx and dy to polar coordinates.

   (b) (**30 pts**) Use (a) to calculate $\int_C dx + dy$ where C is given by
   $$r = \sin\theta, \quad 0 \le \theta \le \pi.$$

   (c) (5 pts) Use the result in (a) to solve for dr and $d\theta$.

4. (20 pts) Let f be a function such that
   $$\nabla f = \left(\frac{1}{2} - \frac{y^2}{2x^2}, \frac{y}{x}\right)$$
   for $x > 0$. Find the function $f(x, y)$ by computing some line integral

along a piece-wise line segment path from $(a, 0)$ to (x, y) with $a > 0$ being some fixed constant. **No credit at all if you only give some function.**