MA26500 FINAL EXAM INSTRUCTIONS Fall 2019

GREEN - Test Version 01

NAME ______________________ INSTRUCTOR ______________________

1. You must use a **#2 pencil** on the mark–sense sheet (answer sheet).
2. On the mark–sense sheet, fill in the **instructor's** name (if you do not know, write down the class meeting time and location) and the **course number** which is **MA265**.
3. Fill in your **NAME** and blacken in the appropriate spaces.
4. Fill in the **SECTION Number** boxes with section number of your class (see below if you are not sure) and blacken in the appropriate spaces.

111	TR	12:00 PM	Meng-Che "Turbo" Ho	155	MWF	9:30 AM	Ping Xu
122	MWF	9:30 AM	Xu Wang	156	MWF	10:30 AM	Ying Chen
132	MWF	8:30 AM	Xu Wang	157	MWF	11:30 AM	Brian Krummel
145	TR	10:30 AM	Theresa Anderson	158	MWF	11:30 AM	Ping Xu
147	TR	9:00 AM	Yating Wang	159	MWF	10:30 AM	Ping Xu
148	TR	1:30 PM	Meng-Che "Turbo" Ho	161	MWF	2:30 PM	Ying Chen
149	TR	9:00 AM	Theresa Anderson	162	MWF	12:30 PM	Brian Krummel
150	MWF	3:30 PM	Ning Wei	163	MWF	1:30 PM	Ning Wei
152	TR	3:00 PM	Xuefeng Xu	172	TR	7:30 AM	Yating Wang
153	TR	4:30 PM	Xuefeng Xu	173	TR	7:30 AM	Marius Vladoiu
154	MWF	8:30 AM	Ying Chen	174	TR	9:00 AM	Marius Vladoiu

5. Fill in the correct TEST/QUIZ NUMBER (**GREEN** is 01).
6. Fill in the 10-DIGIT PURDUE ID and blacken in the appropriate spaces.
7. Sign the mark–sense sheet.
8. Fill in your name and your instructor's name on the question sheets (above).
9. There are 25 questions, each worth 8 points. **BLACKEN** in your choice of the correct answer in the spaces provided for questions 1–25 in the answer sheet. Do all your work on the question sheets, in addition, also **CIRCLE** your answer choice for each problem on the question sheets in case your scantron is lost. **Turn in both the mark-sense sheets and the question sheets to your instructor when you are finished**.
10. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.
11. NO CALCULATORS, BOOKS, NOTES, PHONES OR CAMERAS ARE ALLOWED on this exam. Turn off or put away all electronic devices. Use the back of the test pages for scrap paper.

1. Given the determinant

$$\begin{vmatrix} 2 & d & a+3d \\ 2 & e & b+3e \\ 10 & 5f & 5c+15f \end{vmatrix} = 120,$$

what is the determinant

$$\begin{vmatrix} a & b & c \\ d & e & f \\ 1 & 1 & 1 \end{vmatrix} ?$$

A. -4

B. 12

C. 120

D. -12

E. 4

2. Let A be an $m \times n$ matrix and $\mathbf{b}$ be a non-zero vector in $\mathbb{R}^m$. Which of the following statements must be TRUE?

(i) If $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, then $A\mathbf{x} = \mathbf{b}$ has no solution.

(ii) If $A\mathbf{x} = \mathbf{b}$ has exactly one solution, then A is an $m \times n$ matrix with $m \geq n$.

(iii) If $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions, then $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

(iv) If A is an $n \times n$ square matrix, then $A\mathbf{x} = \mathbf{0}$ has exactly one solution.

A. (ii) only

B. (iii) only

C. (i) and (ii) only

D. (iii) and (iv) only

E. (ii), (iii), and (iv) only

3. Given that $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, what is $(A^{-1}\,\mathbf{b})^T$?

A. $\begin{bmatrix} 12 & -4 & 3 \end{bmatrix}$

B. $\begin{bmatrix} 14 & -5 & 3 \end{bmatrix}$

C. $\begin{bmatrix} 18 & -7 & 3 \end{bmatrix}$

D. $\begin{bmatrix} 20 & -8 & 3 \end{bmatrix}$

E. $\begin{bmatrix} 18 & 7 & 3 \end{bmatrix}$

4. Suppose

$$\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} \right\}$$

is an orthogonal set, and x, y, z are not all zero. Which of the following is TRUE?

A. $x/z = 3$

B. $x/z = 3/10$

C. $y/z = 3$

D. $y/z = 3/10$

E. None of the above

5. Let $L : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation and $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a basis for $\mathbb{R}^3$, where

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}.$$

Suppose that

$$L(\mathbf{v}_1) = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \quad L(\mathbf{v}_2) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad L(\mathbf{v}_3) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

What is the standard matrix for the linear transformation L (relative to the standard bases on $\mathbb{R}^3$ and $\mathbb{R}^2$)?

A. $\begin{bmatrix} -1 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$

B. $\begin{bmatrix} -1 & 0 & -2 \\ 1 & 1 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & -2 \end{bmatrix}$

D. $\begin{bmatrix} -1 & -1 & -2 \\ 1 & 0 & 1 \end{bmatrix}$

E. $\begin{bmatrix} -1 & -1 & -2 \\ 1 & 1 & 1 \end{bmatrix}$

6. Let V be a vector space spanned by vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$. Then which statement is TRUE?

A. $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ must be linearly independent.

B. $\dim(V) = 0, 1, 2, \text{or } 3$.

C. $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ form a basis for V.

D. $\dim(V) = 3$.

E. $\dim(V) < 3$.

7. Find all real numbers k such that the parallelogram with vertices

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ k \end{bmatrix}, \quad \begin{bmatrix} 4 \\ 5 \end{bmatrix}, \quad \begin{bmatrix} 6 \\ k+5 \end{bmatrix}$$

has area 6.

A. $k = -1$ only

B. $k = -1$ or 4

C. $k = 1$ only

D. $k = 4$ only

E. $k = 1$ or 4

8. Suppose $A = \begin{bmatrix} -1 & -3 & 0 & -4 & -6 \\ -1 & -1 & -1 & -1 & -3 \\ 2 & -1 & 2 & -4 & 0 \end{bmatrix}$. Which of the following statements is <u>FALSE</u>?

A. $\left\{ \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} \right\}$ is a basis for $\operatorname{Col} A$.

B. $\left\{ \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -4 \\ -1 \\ -4 \end{bmatrix} \right\}$ is a basis for $\operatorname{Col} A$.

C. $\left\{ \begin{bmatrix} 2 \\ -2 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$ is a basis for $\operatorname{Nul} A$.

D. $\left\{ \begin{bmatrix} -2 \\ 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ is a basis for $\operatorname{Nul} A$.

E. $A\mathbf{x} = \begin{bmatrix} 5 \\ 2 \\ -8 \end{bmatrix}$ is consistent.

9. Let $\mathbb{P}_3$ be the space of all polynomials of degree at most 3. Which of the following sets are subspaces of $\mathbb{P}_3$?

(i) Set of all polynomials $\mathbf{p}$ in $\mathbb{P}_3$ such that $\mathbf{p}(0)\,\mathbf{p}(2) = 0$.

(ii) Set of all polynomials $\mathbf{p}$ in $\mathbb{P}_3$ such that $\mathbf{p}(1) = 4\,\mathbf{p}(0) + 2$.

(iii) Set of all polynomials $\mathbf{p}$ in $\mathbb{P}_3$ such that $\mathbf{p}(1) = 0$ and $\mathbf{p}(4) = 0$.

A. (i) and (ii) only

B. (i) only

C. (i) and (iii) only

D. (ii) and (iii) only

E. (iii) only

10. Suppose A and B are $n \times n$ matrices. Which of the following statements must be TRUE?

A. $AB = BA$.

B. If $AB = I$, then B is the inverse of A.

C. If A and B are invertible, then $(AB)^{-1} = A^{-1}B^{-1}$.

D. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ is invertible.

E. If $A\mathbf{x} = \mathbf{b}$ is consistent for some $\mathbf{b}$ in $\mathbb{R}^n$, then A is invertible.

R1. V vector space,

(*) (subset) $H \subseteq V$. If H is a subspace of V.

① $0 \in H$

② $u_1 \in H$, $u_2 \in H$, $u_1 + u_2 \in H$

③ $u_1 \in H$, $\alpha \in \mathbb{R}$, $\alpha u_1 \in H$.

R2. $\mathbb{P}_3$. std basis: $1, t, t^2, t^3$

zero in $\mathbb{P}_3$: $0 = 0 \cdot 1 + 0t + 0t^2 + 0t^3$

(ii) $H = \{ p,\ p(1) = 4\, p(0) + 2 \}$

$p(t) = a + bt + ct^2 + dt^3$

If $p(t)$ is in H,

$$a + b \cdot 1 + c1^2 + d1^3 = 4 a + 2$$

$$-3a + b + c + d = 2$$

$H = \{ a, b, c, d,\ \text{s.t. } -3a + b + c + d = 2 \}$

H is not a subspace of $\mathbb{R}^4$

b/c, 0 is not in H.

$\Rightarrow$ H is not a subspace of $\mathbb{P}_3$

(ii) $\{ p(t), \ p(1)' = 2 \}$

NO. (0 is not in H)

(iii) $H = \{ p, \ p(1) = 0, \ p(4) = 0 \}$

$p(t) = a + bt + ct^2 + dt^3$

$$\begin{cases} a + b + c + d = 0 \\ a + 4b + 16c + 64d = 0 \end{cases}$$

$\Downarrow$

$$A \cdot \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = 0$$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 4 & 16 & 64 \end{bmatrix}_{2 \cdot 4}$$

null (A) is subspace of $\mathbb{R}^4$

$\Rightarrow$ H is subspace of $\mathbb{P}_3$

(iii)' $\{ p(t),\ p(1) = 0 \}$ Yes.

(i) $\{ p(t) : p(0) \cdot p(2) = 0 \} = H$.

$p(t) = a + bt + ct^2 + dt^3$

$\{ a, b, c, d,\ a\,(a + 2b + 4c + 8d) = 0 \} = H$

I. 0 is in H, $a = b = c = d = 0$

II $a = 0 \quad b = c = d = 1$

$p_1(t) = t + t^2 + t^3 \in H$.

$a = 14 \quad b = c = d = -1$

$p_2(t) = 14 - t - t^2 - t^3 \in H$

$p_3(t) = p_1(t) + p_2(t) = 14$

$a = 14,\ b = c = d = 0 \Rightarrow p_3 \notin H$.

$\Rightarrow$ this is not closed under addition.

$\Rightarrow$ not a subspace.

(i) {all polynomials of degree = 3.}

$= \{ c t^3, c \neq 0 \}$

check if 0 is in H, ⇒ not a subspace.

11. Which of the following statements are/is TRUE?

(i) $\begin{bmatrix} 6 & 1 \\ 5 & 3 \end{bmatrix}$ is in the subspace of 2×2 matrices spanned by $\left\{ \begin{bmatrix} 2 & -1 \\ 3 & -4 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \right\}$.

(ii) The polynomial $1+2t+3t^2$ is in the subspace of polynomials of degree at most three spanned by $\{2t+t^2-t^3, -2t+3t^2, 2t^2-3t^3\}$.

(iii) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} a \\ 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ a \\ -1 \\ 2 \end{bmatrix} \right\}$ is linearly independent for all real values of a.

A. (i) and (ii) only

B. (i) and (iii) only

C. (i) only

D. (iii) only

E. (i), (ii), and (iii)

12. Let A and B be 4×4 matrices such that

$$A = \begin{bmatrix} 2 & 5 & 0 & 4 \\ 3 & 8 & 11 & 5 \\ 4 & 6 & 3 & 8 \\ 2 & 9 & 0 & 4 \end{bmatrix}$$

and $\det(B) = 6$. What is $\det(AB^{-1})$?

A. -24

B. 4

C. 24

D. -4

E. 144

13. Consider the equation $A\mathbf{x} = \mathbf{0}$, where A is a 5×7 matrix. Which of the following statements is/are TRUE?

(i) $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions.

(ii) The matrix A has $\text{rank}(A) \leq 5$.

(iii) The associated linear system has exactly two free variables.

A. (i) and (ii) only

B. (i) and (iii) only

C. (i) only

D. (ii) only

E. (i), (ii), and (iii)

the element in $M_{3\times3}$ is a matrix (vector)

14. Let $\mathbb{M}_{3\times3}$ be the space of 3×3 matrices. Let H be the subspace of $\mathbb{M}_{3\times3}$ consisting of all matrices $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ such that $A \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. What is the dimension of H?

A. 1

B. 3

C. 6

D. 8

E. 9

① dimension: the number of vectors in the basis.

② count the # of "free" variables.

$$A \cdot \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Leftrightarrow \begin{cases} a - 2b + 0 \cdot c = 0 & (1) \\ d - 2e + 0f = 0 & (2) \\ g - 2h + 0 \cdot i = 0 & (3) \end{cases}$$

free variables : $c \quad f \quad i$

(1) $\Rightarrow$ $a = 2b$, b is also free, but a depends b is not free.

(2) $\Rightarrow$ $d = 2e$, e is free, d is not free.

(3) $\Rightarrow$ h - - - - g - - - - -

we 6 free variables $\Rightarrow$ $\dim(H) = 6$. //

Basis of H.

eg null(B) : $s \begin{pmatrix} 1 \\ 2 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $s, t \in \mathbb{R}$.

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

$$= \begin{pmatrix} 2b & b & c \\ 2e & e & f \\ 2h & h & i \end{pmatrix}$$

$$= \begin{pmatrix} 2b & b & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & 0 & 0 \\ 2e & e & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & f \\ 0 & 0 & 0 \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2h & h & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & i \end{pmatrix}$$

$$= b \cdot \begin{pmatrix} 2 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

→ one vector in the basis

$$c \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

basis element.

15. For the system of differential equations $\mathbf{x}'(t) = A\mathbf{x}(t)$ with $A = \begin{bmatrix} 1 & -5 \\ 2 & -6 \end{bmatrix}$, the origin is

A. an attractor

B. a repeller

C. a saddle point

D. a spiral point

E. none of the above

16. Consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} a \\ 1 \\ 2 \end{bmatrix}, \quad T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}, \quad T\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ a \\ 4 \end{bmatrix},$$

where a is a real number. Which of the following statements are/is TRUE?

(i) The linear transformation T is one-to-one for every real value of a.

(ii) The linear transformation is not onto for $a = 2$.

(iii) The standard matrix for the linear transformation T (relative to the standard basis on $\mathbb{R}^3$) has rank 3 for all real numbers $a \neq 2, 8$.

A. (ii) only

B. (i) and (ii) only

C. (iii) only

D. (ii) and (iii) only

E. (i), (ii) and (iii)

17. Find the distance from the vector $\mathbf{y} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$ to the subspace W of $\mathbb{R}^3$ spanned by $\mathbf{u}_1 = \begin{bmatrix} 2 \\ -3 \\ 3 \end{bmatrix}$ and $\mathbf{u}_2 = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$.

A. $\sqrt{2}$

B. 2

C. 4

D. $\sqrt{5}$

E. 5

18. Consider the following system of differential equations

$$\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} = \begin{bmatrix} -7 & 9 \\ -6 & 8 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

with initial condition $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$. What is $x_1(2) + x_2(2)$?

A. $4e^{-1} + 5e^2$

B. $4e^{-2} + 5e^4$

C. $5e^{-2} + 4e^4$

D. $5e^2 + 4e^{-4}$

E. $5e^{-2} + 4e^{-4}$

19. Let $C[-1,1]$ be the vector space of all continuous functions defined on [-1,1]. Define with the inner product on $C[-1,1]$ by

$$\langle f, g\rangle = \int_{-1}^{1} f(t)\, g(t)\, dt.$$

Find the orthogonal projection of $10t^3 - 5$ onto the subspace spanned by 1 and t (with respect to the above inner product on $C[-1,1]$).

A. $6t - 10$

B. $6t + 5$

C. $10t^3 - 6t$

D. $10t^3 - 5$

E. $6t - 5$

20. Which of the following statements is TRUE?

A. If 2×2 matrices A and B have the same eigenvalues (with the same multiplicities), then A is similar to B.

B. Row operations on a square matrix do not change its eigenvalues.

C. There exists an $n \times n$ nonzero matrix A such that every nonzero vector in $\mathbb{R}^n$ is an eigenvector of A.

D. The numbers 1, -1, and 0 can be the eigenvalues of a 3×3 invertible matrix.

E. The matrix $\begin{bmatrix} 4 & -5 \\ 5 & -6 \end{bmatrix}$ is diagonalizable.

21. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 9 \\ -3 & k \end{bmatrix}$. What value of k, if any, will make $AB = BA$?

A. $k = -1$.

B. $k = 0$.

C. $k = 1$.

D. $k = 2$.

E. No value of k will make $AB = BA$.

22. Consider the basis S for $\mathbb{R}^3$ given by

$$S = \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

Applying the Gram-Schmidt process to S produces which orthonormal basis for $\mathbb{R}^3$?

A. $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \frac{1}{5}\begin{bmatrix} -2 \\ 4 \\ 10 \end{bmatrix}, \frac{1}{2}\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\}$

B. $\left\{ \frac{1}{\sqrt{5}}\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \frac{1}{\sqrt{5}}\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

C. $\left\{ \frac{1}{\sqrt{5}}\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \frac{1}{\sqrt{30}}\begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}, \frac{1}{\sqrt{6}}\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\}$

D. $\left\{ \frac{1}{\sqrt{5}}\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \frac{1}{\sqrt{5}}\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \frac{1}{\sqrt{5}}\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

23. Consider the inconsistent linear system $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 1 & 0 \\ -1 & -1 \\ 1 & 2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}.$$

The least-square solution $\widehat{\mathbf{x}}$ of the linear system is

A. $\begin{bmatrix} 1 \\ -3 \end{bmatrix}$

B. $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$

C. $\begin{bmatrix} 8 \\ -6 \end{bmatrix}$

D. $\begin{bmatrix} 2 \\ -6 \end{bmatrix}$

E. None of the above.

24. Let

$$A = \begin{bmatrix} 7 & -6 \\ 1 & 2 \end{bmatrix}.$$

Then the matrix A^{2019} is

A. $\begin{bmatrix} 3 \cdot 5^{2019} - 2 \cdot 4^{2019} & 6 \cdot 4^{2019} - 6 \cdot 5^{2019} \\ 5^{2019} - 4^{2019} & 3 \cdot 4^{2019} - 2 \cdot 5^{2019} \end{bmatrix}$

B. $\begin{bmatrix} 2 \cdot 5^{2019} - 3 \cdot 4^{2019} & 5^{2019} - 4^{2019} \\ 2 \cdot 4^{2019} - 2 \cdot 5^{2019} & 2 \cdot 4^{2019} - 5^{2019} \end{bmatrix}$

C. $\begin{bmatrix} 3 \cdot 5^{2019} + 2 \cdot 4^{2019} & -6 \cdot 4^{2019} - 6 \cdot 5^{2019} \\ 5^{2019} + 4^{2019} & -3 \cdot 4^{2019} - 2 \cdot 5^{2019} \end{bmatrix}$

D. $\begin{bmatrix} 7^{2019} & -6^{2019} \\ 1 & 2^{2019} \end{bmatrix}$

E. $\begin{bmatrix} 2 \cdot 5^{2019} + 3 \cdot 4^{2019} & 5^{2019} + 4^{2019} \\ -2 \cdot 4^{2019} - 2 \cdot 5^{2019} & -2 \cdot 4^{2019} - 5^{2019} \end{bmatrix}$

25. Which of the following orthogonal matrix Q satisfies

$$Q^T \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}?$$

A. $Q = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{3} & 1/\sqrt{2} & 0 \\ 1/\sqrt{3} & 0 & 1/\sqrt{2} \end{bmatrix}$

B. $Q = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{2} & -1/\sqrt{6} \\ -1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & 2/\sqrt{6} \end{bmatrix}$

C. $Q = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

D. $Q = \begin{bmatrix} 1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{6} & 1/\sqrt{6} & 2/\sqrt{6} \end{bmatrix}$

E. $Q = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 1/\sqrt{3} \\ 1/\sqrt{2} & 0 & -1/\sqrt{3} \\ 0 & 1/\sqrt{2} & 1/\sqrt{3} \end{bmatrix}$