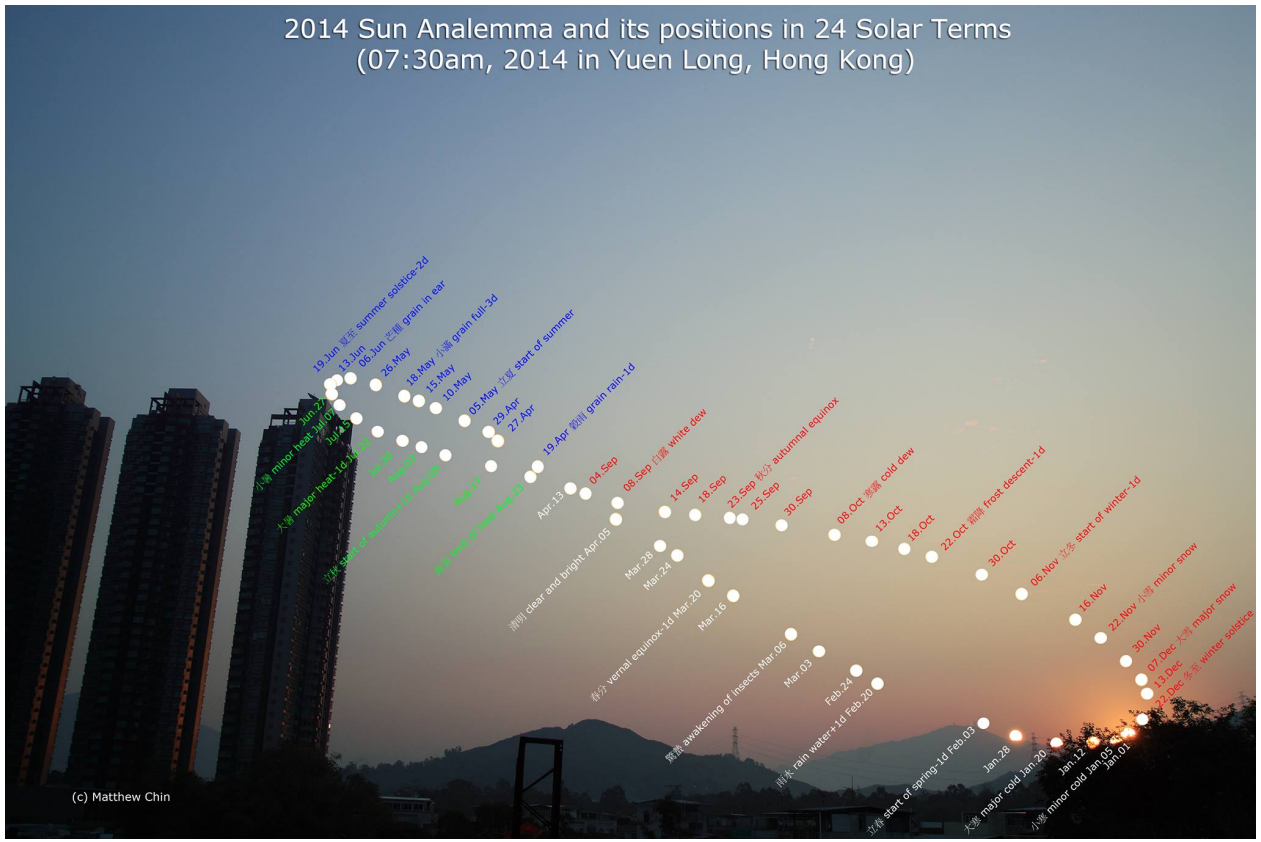
# A Figure Eight Phenonemon in Astronomy

Yilong Zhang

(joint work with Yuchen Zhang)

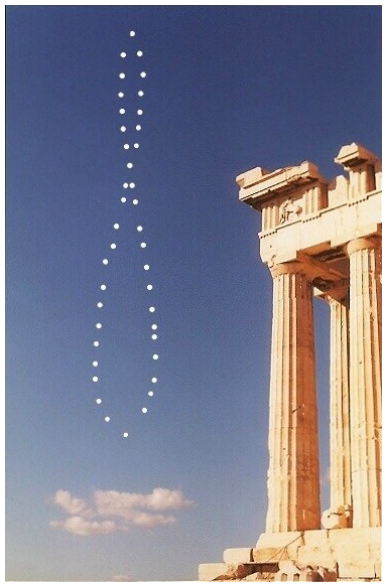
April 16, 2022

# A figure eight in the sky

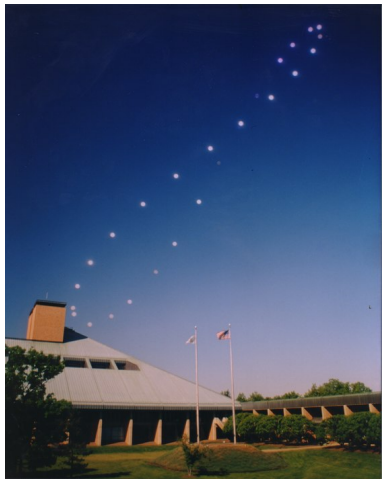



It is a trace of the sun at a fixed time in a day through a year and at a fixed place on the earth. Such a figure is called an *analemma*.

# More figures of analemma



12:28 pm UT+2, Jan 12 - Dec 21, 2002, Parthenon, Athens, Greece

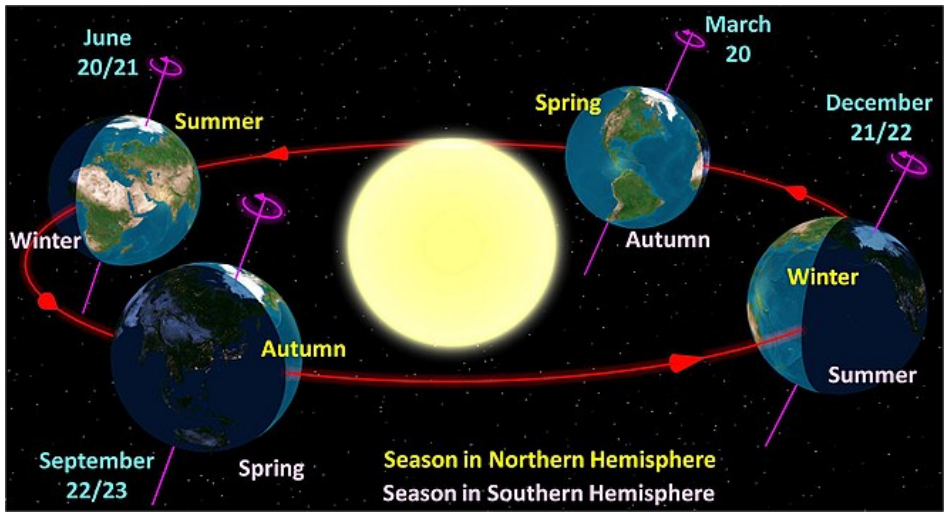


98-99, Murray Hill, New Jersey, USA

# Questions

1. Why the trace of the sun is a figure of eight?
2. How does the figure dependent on the local time and the location of the observer?

# What we know about the earth and sun




- The earth's orbit around the sun is an ellipse (with eccentricity=0.01671) and the sun sits at one of the focus.
- The earth spins around an axis perpendicular to the equator, and the equator tilts 23.5 degrees from the earth-sun orbit.

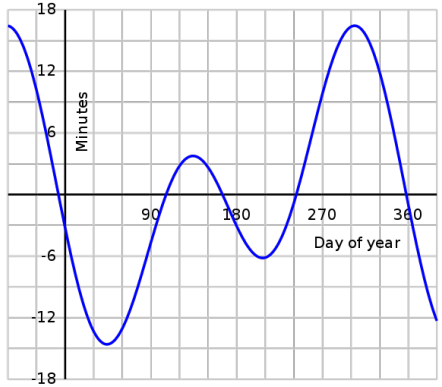
There is a function $y(t) = 23.5^\circ \sin(t - \theta)$ describing the latitude of the "direct sun light" point, where $\theta$ is the phase corresponding to Mar 21.

# Equation of time

The two factors will result in the discrepancy between the "sundial time" and the "clock time". Their difference is called the *equation of time*.

Sundial time

Clock time




Equation of time

The sundial time reflects the position of the actual sun, while the clock time reflects the position of an imaginary sun that moves uniformly 24 hours a day, 365 days a year.

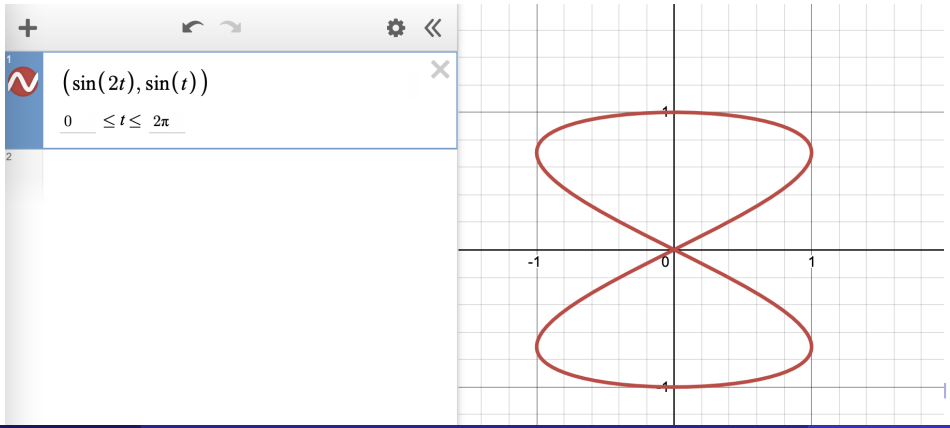
# Equation of time is the horizontal part of figure eight

## Fact

Let $x(t)$ the equation of the time, and $y(t)$ be the function of latitude of the "direct sun point" where $t$ varies in a year, then (x(t),y(t)) describes an analemma observed on the equator of the earth at 12 pm local time.

## Intuition

The equation of time can be regarded as a "$\sin(2t)$"-function with perturbation. $y(t)$ can be regarded as a "$sin(t)$"-function. Naively, an analemma derives from $(\sin(2t), \sin(t))$ has graph looks like a figure of eight.



# Our Results

We answered the original questions:

1. We build a model and computed an approximation of the equation of time.
2. Describe an analemma observed at any latitude $\phi \in [-90°, 90°]$ and any $h \in [0, 24]$.

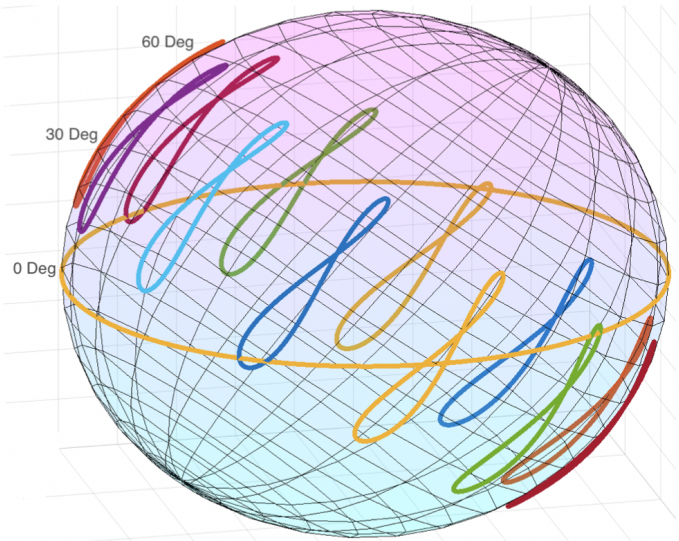


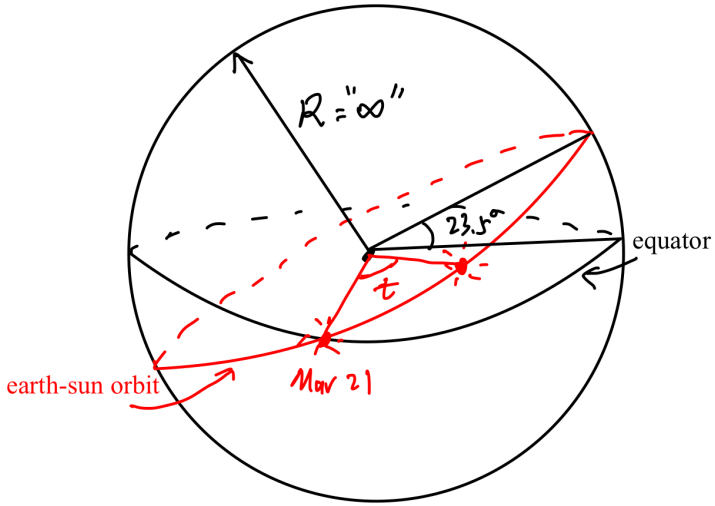

Figure: Analemma at latitude 51.4791° north, longitude 0° in every 2 hours through a day

# Assumptions

- Earth's radius $<<$ earth's distance to sun.
- No gravitational effect from other planets.
- Earth is round; no atmosphere-related optical effect, etc.

# A geocentric model for sun-earth orbit

Instead of thinking of the earth orbiting around the sun, we think the sun orbits the earth relatively. The position of the sun to the sphere "at infinity" is centered at the earth. The sphere is called the *celestial sphere*.




Mar 21 marks one of the two unique days when daytime equals nighttime on the earth. Let $d$ be the number of the days since Mar 21.

$$t = \frac{d}{365.25} \times 2\pi + \text{ellipticity effect.}$$

# The end

To see our detailed solutions and results, please stop by Yuchen's poster.