Problem: Show that the integer nearest to \( \frac{n!}{e} \) \((n \geq 2)\) is divisible by \( n - 1 \) but not by \( n \).

Solution  (by Elie Ghosn, Montreal, Quebec)

We have \( e^{-1} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \). Therefore,

\[
\frac{n!}{e} = n!e^{-1} = n! \sum_{k=0}^{n} \frac{(-1)^k}{k!} + n! \sum_{k=n+1}^{\infty} \frac{(-1)^k}{k!}.
\]

The first term is obviously an integer and the second term can be bounded by (remainder of an alternating series)

\[
\left| n! \sum_{k=n+1}^{\infty} \frac{(-1)^k}{k!} \right| \leq n! \cdot \frac{1}{(n+1)!} = \frac{1}{n+1} \leq \frac{1}{3} \text{ since } n \geq 2.
\]

Therefore \( n! \sum_{k=0}^{n} \frac{(-1)^k}{k!} \) is the nearest integer to \( \frac{n!}{e} \). This integer is not divisible by \( n \) because:

\[
n! \sum_{k=0}^{n} \frac{(-1)^k}{k!} = n \cdot \left[ (n-1)! \sum_{k=0}^{n-1} \frac{(-1)^k}{k!} \right] + (-1)^n
\]

and it is divisible by \((n - 1)\) because:

\[
n! \sum_{k=0}^{n} \frac{(-1)^k}{k!} = n(n-1) \left[ (n-2)! \sum_{k=0}^{n-2} \frac{(-1)^k}{k!} \right] + (-1)^{n-1} \cdot n + (-1)^n
\]

\[
= (n-1) \left\{ n \left[ (n-2)! \sum_{k=0}^{n-2} \frac{(-1)^k}{k!} \right] + (-1)^{n-1} \right\},
\]

since terms between square bracket are obviously integers.

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